This assignment is for students enrolled in MATH2069. It is due before 4pm on Thursday May 3 and must be submitted in an electronic form which is a scanned copy of your answers. The assignment solutions must be uploaded via Turnitin in the Learning Management System (Blackboard). It should include your name and SID. Printed/typed solutions are acceptable. You may quote any result that you need from lectures. All arguments and working must be shown.

1. (a) Solve the recurrence relation

\[ a_n = a_{n-1} + 8a_{n-2} - 12a_{n-3} + 25(-3)^{n-2} + 32n^2 - 64n, \quad n \geq 3, \]

where \( a_0 = 130, \ a_1 = 215 \) and \( a_2 = 260 \).

(b) Write down a closed formula for the generating function of the sequence \( a_n \).

(5 marks)

2. (a) Two sequences \( a_n \) and \( b_n \) are related by the identities

\[ b_n = (-1)^n(n+1)a_0 + (-1)^{n-1}n a_1 + \cdots + (-1)^2 a_{n-1} + a_n \]

which hold for all \( n \geq 0 \). Find an expression for the generating function for the sequence \( b_n \) in terms of the generating function for the sequence \( a_n \).

(b) Use your solution to part (a) to prove the identity

\[ \binom{\alpha}{n} = \sum_{k=0}^{n} (-1)^k(k+1) \binom{\alpha+2}{n-k} \]

for all \( n \geq 0 \), where \( \alpha \) is a complex number.

(5 marks)