This quiz lasts 30 minutes. There are a total of 15 marks.

Answers must be written in **pen** in the boxes provided. Anything written in pencil or written outside the boxes will not be marked.

Your answers can be given in the form of an expression such as $4 \times 3 \times \binom{7}{2}$ or $S(9,5)$, if you wish, as long as it fits in the answer box – i.e. you are not required to find the numerical value.
1. Complete the following definitions and formulations of theorems.
   (a) If \( g : A \to B \) is a function and \( b \in B \), then the preimage \( g^{-1}(b) \) is
   \[
g^{-1}(b) = \{ a \in A \mid g(a) = b \}\]
   (b) If \( n, k \in \mathbb{N} \), then the Stirling number \( S(n, k) \) is defined as
   the number of partitions of the set \( \{1, \ldots, n\} \)
   into a disjoint union of \( k \) nonempty subsets.
   (c) The Binomial Theorem gives the expansion
   \[
   (x + y)^m = \sum_{k=0}^{m} \binom{m}{k} x^{m-k} y^k
   \]
   (3 marks)

2. Count the subsets of the set \( \{ n \in \mathbb{Z} \mid 1 \leq n \leq 70 \} \)
   (a) of cardinality 7
   \[
   \binom{70}{7}
   \]
   (b) of cardinality 7 whose elements are multiples of 6
   \[
   \binom{11}{7}
   \]
   (c) of cardinality 7 containing exactly 5 odd numbers
   \[
   \binom{35}{5} \binom{35}{2}
   \]
   (3 marks)
3. How many ways are there to put 13 different coins in 6 boxes $B_1, \ldots, B_6$ so that

(a) no box is left empty
(b) only two
of the six boxes are left empty
(c) each box
contains at least two coins

$$6! S(13, 6)$$
$$\binom{6}{2} 4! S(13, 4)$$
$$6 \binom{13}{3, 2, 2, 2, 2, 2}$$

(3 marks)

4. Count the number of sequences of digits 1, 2, 3 of length 10

(a) which increase weakly from left to right

$$\binom{12}{10} = 66$$

(b) which increase weakly from left to right and
each of the digits occurs at least once

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(2 marks)
5. Consider the recurrence relation \( b_n = -6 b_{n-1} - 9 b_{n-2} \), for \( n \geq 2 \).
   
   (a) Write down its characteristic polynomial:
   
   \[
   x^2 + 6x + 9
   \]

   (b) Write down the general solution:
   
   \[
   b_n = C_1(-3)^n + C_2 n (-3)^n \text{ for some constants } C_1, C_2
   \]

   (c) Give the solution when \( b_0 = 1 \) and \( b_1 = 0 \):
   
   \[
   b_n = (-3)^n - n(-3)^n
   \] (3 marks)

6. Consider the recurrence relation \( a_n = r_1 a_{n-1} + r_2 a_{n-2} + r_3 a_{n-3} \), for \( n \geq 3 \). The roots of its characteristic polynomial are \( x = -2 \) of multiplicity 2 and \( x = 3 \) of multiplicity 1. Write down the general solution:

   \[
   a_n = C_1 (-2)^n + C_2 n (-2)^n + C_3 3^n \text{ for some constants } C_1, C_2, C_3
   \] (1 mark)

END OF QUIZ.