For parts (a) and (b), see the end of [Dub2009, Section 1.4: *Dressing operator*].

On the imaginary axis, the coefficients $a(k)$ and $b(k)$ are real, since the corresponding eigen-functions of the Schrödinger operator will be real. Set $a_r(\kappa) := a(i\kappa)$, for $\kappa > 0$. Then $a_r'(\kappa) = ia'(i\kappa)$, so (b) implies that the sign of b_s equals to the sign of $a_r'(\kappa_s)$.

We proved in the lectures that $a(k) \sim 1 + O(1/k)$ as $|k| \to \infty$, therefore $a_r'(\kappa_1) > 0$, thus $b_1 > 0$.

The signs of $a_r'(\kappa_s)$ alternate since the zeroes κ_s are simple, which concludes the proof.

References

[Dub2009] B. A. Dubrovin, *Integrable systems and Riemann surfaces*, 2009. lecture notes, available at the author's web-page.