

Integrable Systems: Assessment 3

AMH2: Applied Mathematics Honours

Semester 2, 2017

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Due 9am Tuesday 29 August 2017

Submit scanned or typeset answers to TurnItIn on LMS Blackboard.
Bring your hardcopy to Lecture at 2pm.

1. Prove that:

(a) $\int_{-\infty}^{\infty} \varphi^2(x, i\kappa_s) dx = ia'(i\kappa_s)b_s$;

(b) the zeroes of $a(k)$ are simple;

(c) $(-1)^{s-1}b_s > 0$.

[*Hint*: Start by differentiating the relation $L\varphi(x, k) = k^2\varphi(x, k)$ with respect to k at point $k = i\kappa_s$, then multiply the obtained relation by $\varphi(x, i\kappa_s)$, integrate over the real x -axis, and apply twice integration by parts.]