

Integrable Systems: Assessment 4

AMH2: Applied Mathematics Honours

Semester 2, 2017

Lecturer: *Nalini Joshi* (Rm 629), *Milena Radnovic* (Rm 624), *Yang Shi* (Rm 633)

Due 9am Tuesday 5 September 2017

Submit scanned or typeset answers to TurnItIn on LMS Blackboard.
Bring your hardcopy to Lecture at 2pm.

1. Find the Jost function solutions ϕ , ψ and $\bar{\psi}$ of the stationary Schrödinger equation

$$\psi_{xx} + (6 \operatorname{sech}^2(x) + \zeta^2) \psi = 0$$

and hence find $a(\zeta)$ and $b(\zeta)$ explicitly. Where is $a(\zeta)$ analytic in the complex ζ -plane?

[*Hint:* You may find it useful to transform variables to $z = \tanh x$. Compare the resulting ODE with the one for Associated Legendre functions in any book on special functions. A good online resource is the Digital Library of Mathematical Functions: <http://dlmf.nist.gov/> .]