

Integrable Systems: Assessment 6

AMH2: Applied Mathematics Honours

Semester 2, 2017

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Due 9am Tuesday 19 September 2017

Submit scanned or typeset answers to TurnItIn on LMS Blackboard.
Bring your hardcopy to Lecture at 2pm.

1. Below, we use the terminology for Darboux transformations used in lectures. Refer to the lecture notes for the definition of \tilde{y} .

- (a) Find the Darboux transformation which maps the Schrödinger equation

$$y'' + (n(n+1)\operatorname{sech}^2(x) - \lambda)y = 0,$$

to

$$z'' + ((n+1)(n+2)\operatorname{sech}^2(x) - \lambda)z = 0,$$

where primes denote derivatives in x .

- (b) For what value of $\tilde{\lambda}$ does \tilde{y} satisfy

$$\tilde{y}'' + (n(n+1)\operatorname{sech}^2 x - \tilde{\lambda})\tilde{y} = 0 ?$$

- (c) Consider the equation

$$y''_{m,n} + (n(n+1)\operatorname{sech}^2(x) - m^2)y_{m,n} = 0.$$

Show that this equation has solutions $y_{n,n} = \operatorname{sech}^n(x)$, $y_{n,n-1} = \cosh^n(x)$ and therefore, find $y_{1,2}(x)$.