APPLIED MATHEMATICS 4

2010

THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS AND STATISTICS

Contents

1 Entry Requirements

Entry to the Honours Programme in Applied Mathematics is usually based on satisfying the following (and subject to approval by the Head of the School of Mathematics & Statistics):

1. Faculty requirement: The candidate must have qualified for the Pass Degree with a AAM (Average Annual Mark for second and third year units) of at least 65 %.

2. Mathematics requirement: The candidate must have completed 24 credit points of senior (third-year) mathematics with a Pass Degree average of at least 70 %.

3. Essay / Project supervision: The candidate is expected to find a prospective supervisor from among the Applied Mathematics staff, who is agreeable to supervise the candidate's essay or project in the candidate's chosen topic.

Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above. There is some flexibility in these conditions; students not quite meeting them, but keen to pursue Honours in Applied Mathematics are invited to contact the Course Coordinator for advice. Students are expected to meet with prospective supervisors to discuss the potential for essays or projects before submitting the honours application.

Applications must be submitted to the Faculty of Science.

Application and enrolment information should also be obtained from the Faculty of Science, either in person of from their website. The Faculty will also provide AAM computations once the final third-year results are in.

Graduate Diploma in Science and MSc(Qualifying) applicants should see the Director of Postgraduate Studies before enrolling.

2 Course Administration

The Course Coordinator for Applied Mathematics 4 in 2010 is:

Dr. Martin Wechselberger
Room 628, Carslaw Building
Phone: (02) 9351 3860
Email: `wm@maths.usyd.edu.au`

3 Structure of Applied Mathematics 4

Full-time students normally attend three lecture courses each Semester, for a total of six courses. All 6 courses will count towards the student's final assessment.

In addition to the courses, each student is also required to write an essay or project on an Applied Mathematics topic, under the supervision of a member of staff of the School of Mathematics and Statistics. This is considered to be the major aspect of the Honours Programme, and is detailed in Section 6.

The primary choice for the six courses come from the Applied Mathematics courses we offer each year. Each such course runs for the first 12 weeks of each semester, at 2 lectures per week. There are usually no formal tutorials, but lecturers are happy to help students with their questions by arrangement.

The following Applied Mathematics 4 courses are expected to be offered this year:

First Semester

Advanced Option Pricing	A/Prof C.-O. Ewald
Multiscale Methods with Applications	A/Prof G. Gottwald
Modern Asymptotics and Perturbation Theory	Prof N. Joshi

Second Semester

Geometric Mechanics	A/Prof H. Dullin
Magnetohydrodynamics and Dynamo Theory	Dr. D. Ivers
Interest Rate Models	Prof M. Rutkowski
Mathematical Physiology	Dr M. Wechselberger

Students are also welcome to choose any number of courses from Pure Mathematics 4 or Mathematical Statistics 4, provided that the student's supervisor is happy that the student is covering the proper material required for the student's project or essay. (Details of the available courses, and any entry requirements, should be obtained from the Fourth-Year Course Coordinators in Pure Mathematics and Statistics.)

Students also have the option of choosing a few courses from the following, *for which approval from the Essay / Project Supervisor and the Course Coordinator needs to be obtained prior to enrolling*:

- Fourth-Year Courses from related disciplines such as Physics;
- Third-Year Courses at the Advanced level offered by the School of Mathematics & Statistics;
- Access Grid Room (AGR) Courses offered from other Australian Universities; see `http://www.maths.usyd.edu.au/u/UG/accessgrid.html` for further information;

4 Course Summaries

Advanced Option Pricing *(Sem-1)* *A/Prof C.-O. Ewald*

Assessment: 40% from exam, 30% midterm exam, 30% from one assignment

Financial institutions all around the world employ mathematicians who have had training in the subject of Financial Mathematics. They need to do this because of the large numbers of options and financial derivatives which are traded in modern financial markets. It is by no means an easy task to price these products and that is why most of the institutions have set up quantitative analysis departments that are largely staffed by mathematicians.

We explore in this course all the main theoretical pricing methods which are at the forefront of option and derivative security modelling. Financial asset prices are driven by stochastic models called Wiener processes. The prices of options and derivatives on these assets are obtained through the assumption of arbitrage-free markets. Taken together with Ito's Lemma, this leads to a second-order parabolic partial differential equation for the price of any option or derivative. Solutions of this equation are explored in detail for many types of financial securities including: calls and puts, forward and futures, barrier options, look-back options, bonds and other interest-rate securities, and various selections of exotic options. The modern theory of option pricing is also covered in the course and includes the concepts of: self-financing, replicating portfolios and martingale measures, which ultimately lead to the celebrated Black-Scholes formula.

A good understanding of stochastic processes is an essential requirement of mastering the technical aspects of the course. Thus a sizable part of the course is devoted to this subject and includes topics such as: stochastic differential equations and the Ito stochastic calculus, conditional expectation and martingale representations, transition pdf's and Kolmogorov's diffusion equations, change of measure through the Radon-Nykodym and Girsanov theorems, and the Feynman-Kac formula.

The Fourth Year Honours course is an extension of the Second and Third Year Financial Mathematics units.

Multiscale Methods with Applications *(Sem-1)* *A/Prof G. Gottwald*

Assessment: 50% from exam, 25% from each of two assignments

Most physical and biological systems are too complex to allow for an analytical treatment and very often even for a numerical treatment. This is due to the high dimensionality of the underlying systems and the occurrence of wide ranges of spatial and temporal scales. However, often one is not interested in the description of the full system but rather in some 'interesting' subspace. For example, in numerical weather forecasting one is interested in large scale phenomena such as high and low pressure fields but not in all the small-scale effects such as turbulent wind gusts in your back garden. In the case of time-scale separation and weak coupling, one can try to find the dynamics on a reduced subspace which describes only the dynamics of the 'interesting' dynamics. But we need to know how to model the accumulative effect of all these 'uninteresting' scales.

We will discover very general mathematical tools which allow us to describe the reduced dynamics on some approximate manifold. In this course we will explore how certain stochastic systems and high-dimensional deterministic systems can be effectively reduced to low-dimensional stochastic dynamical systems. We will investigate how to treat so-called unresolved scaled and how to introduce coarse-graining. We first learn standard deterministic methods such as centre-manifold theory and variational asymptotics before we start stochastic model reduction.

Modern Asymptotics and Perturbation Theory *(Sem-1)* *Prof N. Joshi*
Assessment: 60% from take-home exam, 40% from two assignments

Differential equations model most natural phenomena we know. Yet their solutions can be notoriously difficult to understand. In place of "exact" solutions, a rich array of *asymptotic* methods have been developed to qualitatively understand the solutions. These are based on either the intrinsic variables or an external parameter in the problem being large or small. The field is vast and ranges from the fundamental theory of asymptotic expansions and perturbation methods, developed by Poincaré for the study of the solar system, to modern advanced techniques which deal with cases where conventional asymptotics fails. This course will include asymptotics of boundary layers, WKB and multiscale methods and modern techniques developed to model dendritic growth (such as snowflakes), fluid flow (existence of solitary waves), and the onset of chaos. The only background needed is the basic theory of differential equations and complex analysis.

Geometric Mechanics *(Sem-2)* *A/Prof H. Dullin*
Assessment: 60% from exam, 40% from assignments

Classical mechanics still holds some of the most challenging unsolved problems in dynamical systems, like the questions of the stability of the solar system, or even the stability of the idealised three body problem. Geometric Mechanics takes Classical Mechanics to the next level by formulating it on manifolds. The geometric point of view brings new insights into old problems. In this course we will not only study this geometric viewpoint, but also adapt the toolbox of general dynamical system to mechanics. Special attention will be given to integrable systems and their momentum maps. Examples that will be used throughout the course include the n-body problem, the rigid-body with (and without) a fixed point, and geodesic flows.

The only background needed is the basic theory of Lagrangian and Hamiltonian systems.

Magnetohydrodynamics and Dynamo Theory *(Sem-2)* *Dr D. Ivers*

Assessment: 50% from assignment; 50% from exam

Magnetohydrodynamics (mhd) is a union of the hydrodynamics of electrically-conducting fluids and pre-Maxwell electrodynamics (i.e. there is no displacement current, so no electromagnetic waves). Interesting new phenomena arise such as Alfven waves. Applications are drawn from: ideal and resistive mhd, in particular equilibrium & stability in the magnetic confinement problem; rotating mhd. Laboratory, geophysical and astrophysical applications are considered.

The most important application is Dynamo Theory: the self-exciting regeneration of a magnetic field by the motion of an electrically-conducting fluid. The main magnetic fields of the Earth and the planets, Mercury, Jupiter, Saturn, Uranus, Neptune, and possibly the satellite Ganymede, the Sun and other magnetic stars, and perhaps even galaxies, are generated in this way. The emphasis is on the geodynamo and the solar dynamo, which are the best observed and most studied. The ocean and ionospheric disturbance dynamos are also briefly treated.

Interest Rate Modelling *(Sem-2)* *Prof M. Rutkowski*

Assessment: 70% from exam, 30% from three assignments

The fixed-income market is an important sector of the global financial market on which various interest-rate-sensitive instruments, such as: bonds, swaps, swaptions, caps, etc. are traded. The management of interest rate risk, by which we mean the pricing and hedging of interest rate derivatives, is an important and complex issue. The course provides an overview of various concepts related to interest rates, and deals with the most important interest rate-sensitive contracts, such as: interest rate swaps, bond options and swaptions. The crucial part of the syllabus is stochastic modelling of various kinds of interest rates and the valuation of interest rate derivatives within alternative frameworks. We deal with classical examples of short-term rate models, the Heath-Jarrow-Morton methodology, and recently developed market models, such as, the BGM model of LIBORs and Jamshidian's model of forward swap rates.

Relation to other mathematics courses: This course belongs to the field of Financial Mathematics and builds on the material from Probability and Stochastic Processes. Students are expected to have basic knowledge of Itô's stochastic calculus and principles of arbitrage pricing of financial derivatives. The course AMH4 *Advanced Option Pricing* (or equivalent) should be seen as a prerequisite.

Textbooks: Lectures will be based on the course notes. Some chapters from the book: Marek Musiela and Marek Rutkowski: *Martingale Methods in Financial Modelling.* 2nd edition. Springer, 2004, will be recommended for additional reading.

Mathematical Physiology *(Sem-2)* *Dr M. Wechselberger*

Assessment: 60 % from exam/project, 40 % from two assignments

Physiological rhythms are central for life. Prominent examples are the beating of the heart, the activity of neurons, or the release of hormones regulating growth and metabolism. All these rhythms have in common, that they evolve on at least two different time scales, i.e. there exist a quasi steady state of the system on a slow time scale (e.g. the resting state of the heart) interspersed by a dramatic change of the system on a fast time scale (e.g. the heartbeat itself). Mathematical models of such systems are called slow/fast systems or multiple scales problems.

In this unit we introduce a mathematical technique suitable for analysing such multiple scales problems called geometric singular perturbation theory. This perturbation theory is based on the fact that the system under study has a singular perturbation parameter and classical asymptotic analysis breaks down. The method is very powerful and is based on dynamical systems techniques like bifurcation theory and invariant manifold theory. We will develop the basic mathematical tools to analyse physiological problems.

The class of physiological problems we study is electrical signaling in excitable cells. We analyse the famous Hodgkin-Huxley model of the squid giant axon. We use reduction techniques introduced by FitzHugh to obtain a qualitative model of the neuron which can be fully analysed by geometric singular perturbation theory. This analysis will demonstrate how neurons can fire action potentials and therefore communicate information. Another problem of interest is electrical signaling in pancreatic cells which leads to secretion of insulin due to bursting electrical activity.

The only background needed is the basic theory of differential equations and bifurcation analysis (introduced in e.g. MATH3963).

5 Assessment Procedures

The Honours mark for each student is computed based on the following:

- 40 % for the Project/Essay assessment;
- 60 % for 6 courses (10 % for each).

Students are required to attend 6 courses during the academic year and all results will be included in the overall assessment. The assessment procedure for the Project/Essay is outlined in Section 6.

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The Essay/Project, in addition to all the fourth-year courses, will be marked with this scale in mind. This scale appears below.

Grade of Honours	Faculty-Scale
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

Note: All assessable student work (such as assignments, Honours essays and projects) should be completed and submitted by the advertised date. If this is not possible, approval for an extension should be sought *in advance* from the lecturer concerned or (in the case of Honours essays and projects) from the Course Coordinator. Unless there are compelling circumstances, and approval for an extension has been obtained in advance, late submissions will attract penalties as determined by the Board of Examiners (taking into account any applications for special consideration).

Appeals against the assessment of any component of the course, or against the class of Honours awarded, should be directed to the Head of School.

6 The Essay/Project

A significant part of the Honours year is the completion of an Honours Essay or Project by each student. There is a distinction between an essay and a project. A project involves intensive research, analysis or computation and normally requires a greater level of supervision than an essay. An essay may cover a classical problem of acknowledged importance and mathematical depth with the student providing his/her own critical evaluation.

Each student must choose an Essay/Project supervisor who is willing to supervise the student's chosen topic for the Essay or Project. The supervisor must be a member of the staff of the School of Mathematics and Statistics. A list of available topics appears in Section 6.4. However, student are welcome to choose different topics, provided that they are able to obtain a supervisor for that topic from within the School. Essay/project topics and supervisors should be finalised by the beginning of the First Semester, so that students can commence work immediately on their Essays/Projects.

The following list shows the main applied mathematics resarch areas:

- Astrophysical and Geophysical Modelling
- Financial Mathematics
- Industrial Modelling
- Mathematical Biosciences
- Nonlinear Systems

For detailed information about these areas and the corresponding staff, please have a look at the webpage
`http://www.maths.usyd.edu.au/res/AppMaths.html`

6.1 Assessment

The Essay or Project will be marked according to the following.

- 90 % for the final written report
  This will be marked by 3 different markers, one of whom is the supervisor, and each marking will therefore constitute 30 % of the final Essay/Project mark. Note that the assessment also includes a one page report submitted at the end of the first semester (see section 6.3).

- 10 % for a seminar presentation on the Essay/Project

Three typed and bound copies of the final Essay/Project should be submitted to the Applied Mathematics Honours Course Coordinator, who will then distribute these copies to three markers (one of which is the supervisor) for marking. Due dates for submission appear in Section 6.2.

The seminar is an opportunity for each student to present the material of his or her Essay or Project to a mathematically literate audience. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The Course Coordinator will provide additional information and help Honours students in their preparation for the seminar. The presenter of the best AM4 seminar will be awarded the Chris Cannon Prize.

6.2 Important Dates

The following are important dates for all students intending to complete their essay/projects by the end of second semester of 2010 . These deadlines are therefore applicable to most students.

- **Seminar: Week of 13-17th September, 2010 (week 8)**

- **Essay Submission: 10.00 a.m. on Friday, 22rd October 2010 (week 12)**
  Three typed and bound copies of the Essay/Project are to be handed in to the Applied Mathematics Honours Course Coordinator by this date and time.

6.3 Essay/Project Guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the essay or project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report has to be submitted to the Honours coordinator. This report includes a half page description about the students aim/scope of the project/essay and a half page description about what the student has achieved in semester 1 and what the student wants to achieve in semester 2. This report has to be approved by the supervisor before submission.
- The essay/project should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand, though he/she need not be an expert in the field covered. The work must contain substantial mathematical content.
- The essay/project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- Original research is not essential.
- The length of the essay/project should be between 40 to 60 typed A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the essay exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.
- Students should be careful to provide full and correct referencing to all material used in the preparation of essays and projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless reinforcing an important point.
- Three examiners will be appointed to assess each essay/project. One of these examiners will be the student's supervisor. Although marking schemes may differ, marks will generally awarded for:

  (i) selection and synthesis of source material;
  (ii) evidence of understanding;
  (iii) evidence of critical ability;
  (iv) clarity, style and presentation;
  (v) mathematical and/or modelling expertise.

- Students are advised to read the pamphlet entitled "*Guide to Essay Writing for Science Students*" available from the Science Faculty Office.
- The preferred method of typesetting mathematical documents these days is using LaTeX. This is available from the computers at the School. Students are recommended to use LaTeX in typesetting their Essays/Projects. Additional information on LaTeX is available from the Course Coordinator.
- Students who have worked on their essays or project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their essay/project.

6.4 Suggested Topics for the Essay/Project

The following is a list of possible essay/project topics for Applied Mathematics 4 students in 2010 . Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible.

However, this list is not exhaustive. Students may wish to suggest their own topics for essays or projects. Before commencing work, however, each student must find a member of staff who will agree to supervise the essay/project. For topics other than those listed below, the student and supervisor must submit a brief written outline of the proposed project or essay for approval by the Course Coordinator.

The Kerr metric in general relativity (Essay)
Supervisor: Dr C.M. Cosgrove (Carslaw room 716; phone 9351-3357)
The Kerr metric is an exact solution of Einstein's gravitational field equations describing a stationary rotating black hole. Found by Roy Kerr in 1963, it is one of the most celebrated of the exact solutions of Einstein's equations. An essay on this subject would involve deriving the Kerr solution, and discussing its interpretation as a rotating black hole, its horizons, ergosphere, particle and light ray orbits, and analytic extensions.

Gravitational collapse (Essay)
Supervisor: Dr C.M. Cosgrove (Carslaw room 716; phone 9351-3357)
An essay on this subject would begin with a survey of stellar evolution, especially the history of massive stars, showing how white dwarfs, neutron stars and black holes may be formed, and observational evidence for these objects. The mathematical component of the essay would come from a derivation of the Schwarzschild black hole solution of Einstein's equations and some of its consequences, and a discussion of the Oppenheimer-Snyder collapsing dust solution.

Studies in integrable differential equations (Essay or Project)
Supervisor: Dr C.M. Cosgrove (Carslaw room 716; phone 9351-3357)

An essay or project on this topic would focus on some aspect of the theory of integrable ordinary or partial differential equations. Essay topics include group-invariant reductions and nonclassical reductions of partial differential equations. Project topics include Painlevé classification problems (see Ince, "Ordinary Differential Equations," Chapter 14), equivalence problems, inverse scattering and isomonodromy problems.

The Back Drop in Trampolining (Project)
Supervisor: A/Prof H. Dullin (Carslaw room 714; phone 9351 4083)

The back drop is a figure in trampolining in which the athlete takes off straight into the air (with zero angular momentum) and lands rotated by 90 degrees on the back. A simple model of the athlete consists of n planar coupled rigid bodies. The shape of the system is specified by all $n-1$ relative angles. The configuration space is the shape space plus one additional angle describing the overall orientation of the athlete in space. The goal of the project is to describe the shape changes at zero angular momentum that maximise the resulting change in overall orientation.

Simulating a Closed Chain of Planar Rigid Bodies (Project)
Supervisor: A/Prof H. Dullin (Carslaw room 714; phone 9351 4083)

A chain of planar rigid bodies is a simple mechanical system with n segments connected by joints that allow free rotation. Connecting the first segment to the last by another joint gives a closed chain. Since the distance between the joints is fixed the closed chain has n degrees of freedom. Reduction by translations and rotations leaves $n-3$ degrees of freedom specifying the shape. The goal of the project is the solution of Hamilton's equation for the case of the four-gon ($n = 4$), and the simulation of the case of the five-gon using symplectic integration techniques.

A shape changing n-gon (Project)
Supervisor: A/Prof H. Dullin (Carslaw room 714; phone 9351 4083)

Consider an n-gon in the plane all of whose sides have length 1. The relative angles between adjacent sides are allowed to change. Denote the corners of the n-gon by the corresponding points z_i in the complex plane. Let the angles change in such way that $\sum_i \Im(\bar{z}\dot{z}) = const$ and $\sum_i z = 0$. The goal of the project is to describe those periodic changes in the relative angles for which the amount of rotation of the n-gon after one period is maximal. The first interesting case is $n = 5$.

Parameter study of ABC dynamos (Project)
Supervisor: Dr D.J. Galloway (Carslaw room 712; phone 9351-2968)

The family of periodic chaotic flows with velocity fields which are given by $U = (A\sin z + C\cos y, B\sin x + A\cos z, C\sin y + B\cos x)$ is known to be very effective in producing dynamos which generate magnetic fields in electrically conducting fluids. These flows are chaotic, and most studies have concentrated on the case where $A = B = C = 1$ (this has the most symmetries). Given that too much symmetry is often bad for dynamo action, this project proposes the study of a range of other values, using existing linear and nonlinear computational codes which will be supplied.

Chaos in the solar system (Essay or Project)
Supervisor: Dr D.J. Galloway (Carslaw room 712; phone 9351-2968)

The long term stability of the solar system is a problem which has recently been rediscovered because of the growth in the study of chaotic nonlinear dynamical systems. This project could either take the form of a survey of the whole field, in which case it would be an essay, or it could select a particular set of numerical calculations and attempt to reproduce and extend them, giving rise to a project.

Solar prominences (Essay)
Supervisor: Dr D.J. Galloway (Carslaw room 712; phone 9351-2968)

Solar prominences consist of magnetically levitated dense material held aloft in the solar atmosphere by a cradle of magnetic field lines. The study of the existence and stability of these features is an intriguing mathematical problem given additional interest because of the beautiful and detailed images now available from modern space observations. This essay will review existing theories and discuss how well they fare when confronted with observations.

Critical pulse propagation in excitable media (Project)
Supervisor: A/Prof G. Gottwald (Carslaw room 625; phone 9351-5784) Many systems in biology and chemistry are so-called excitable media. Examples are nerve fibres and heart tissue. Excitable media can support wave-trains and spiral waves. Propagation failure of wave-trains is often associated with clinical conditions such as atrial fibrillation in cardiology. The proposed project investigates propagation failure in 1D using a novel perturbation technique. This work is both of analytical and of computational nature. The proposed work will look at issues of propagation failure which have so far not been explored. Requirements: Sound analytical skills, programming experience (preferably in C).

Networks of coupled oscillators (Project)
Supervisor: A/Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Many biological systems are structured as a network. Examples range from microscopic systems such as genes and cells, to macroscopic systems such as fireflies or even an applauding audience at a concert. Of paramount importance is the topography of such a network, ie how the nodes, let's say the fireflies, are connected and how they couple. Can they only see their nearest neighbours, or all of them. Are some fireflies brighter than others, and how would that affect the overall behaviour of a whole swarm of fireflies? For example, the famous 'only 6 degrees of separation'-law for the connectivity of human relationships is important in this context. In this project we aim to understand the influence of the topography of such a network. Question such as: How should a network be constructed to allow for maximal synchronization will be addressed. This project requires new creative ideas and good programming skills.

Cardiac alternans (Project)
Supervisor: A/Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Cardiac alternans is the phenomenon where a short heart-beat duration is followed by a long one, followed by a short one and so forth. It is widely believed that these alternans are the precursors of fatal cardiac failures such as atrial fibrillation. This project - aimed at students interested in mathematical biology - will investigate different models of the heart, and study the possibility of alternans within these models. Questions such as "Are alternans a real instability, and on which physiological time scale does the possible instability evolve?", or "Is this a stable phenomenon?" will be investigated. This project is mainly computational, although in the case of quick progress, we can build an analytical toy-model which captures all the effects observed during the numerical simulations.

Discrete stochastic processes and their asymptotic limits (Project)
Supervisor: A/Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

A reliable and efficient method to distinguish between chaotic and non-chaotic behaviour in noise-contaminated but essentially deterministic data has far reaching applications. For example, the onset of atrial fibrillation is accompanied by non-chaotic electrocardiogram data, hence the ability to detect periodicity is of great clinical significance. Recently, we proposed a new method to detect chaos which applies directly to the time series data and does not require complicated phase space reconstruction. Moreover, the dimension and origin of the dynamical system are irrelevant. The input is the time series data and the output is zero or one depending on whether the dynamics is non-chaotic or chaotic. This zero-one test for chaos is universally applicable to any deterministic dynamical system, in particular to ordinary and partial differential equations. In this project you aim to understand the influence of the sampling time on this test. The sampling time is the time between two distinct time series points - it is like the time between two measurements. In the test you try to detect Brownian motion. However, we are dealing here with a discrete approximation of Brownian motion. How does the approximation affect the predictions, such as diffusion coefficients? For example, it is obvious, that the sampling time has to be small enough. Otherwise, the asymptotic formulae do not make sense anymore. However, what if the sampling time gets smaller and smaller? The theory underlying the test for chaos states, that in the chaotic case, an auxiliary variable will behave asymptotically like Brownian motion. What does asymptotically really mean in this context? In this project you try to understand whether we can quantify the optimal range of sampling times. Discrete Brownian motion as the ones you will study in this project also occurs in the analysis of financial data. This project requires new creative ideas and good programming skills. Knowledge of stochastic calculus and Brownian motion would be an advantage.

Core surface motions (Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

Motions at the surface of the Earth's liquid core can be (partially) determined using the magnetic field measured at the Earth's surface. Various assumptions have been used to uniquely determine the motion including steady flow and tangentially geostrophic flow. This project would involve reviewing the theory underlying core-surface motions and these assumptions and the determination of the flow using one assumption and a particular model of the magnetic field.

Thermal and magnetoconvection (Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

Thermal convection and magneto-convection are important processes in the cores of the Earth, planets and their satellites. This project would involve numerically solving eigenproblems or the steady problem for some particular spherical model using existing codes.

Planetary and galactic dynamo theory (Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

Mercury, Earth, Jupiter and two of its moons, Saturn, Neptune and Uranus have magnetic fields generated by the motions of electrically conducting cores. Stars and galaxies may also possess magnetic dynamos. This computational project would involve an aspect of dynamo theory and the use of existing time-stepping or eigen-/critical- value codes.

P and HP finite element methods (Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

Pseudo-spectral methods offer high accuracy in the numerical solution of partial differential equations but do not offer adaptive refinement, dynamic gridding or simple application to domains of irregular shape. Standard (h-) finite-element methods offer adaptive refinement, dynamic gridding and simple application to domains of irregular shape but low accuracy. P- and hp-finite element methods use higher-order elements and offer significant improvements for singular perturbation problems. This project will implement one-dimensional methods and briefly look at the two-dimensional case.

Protein folding (Essay)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

The genetic code mapping base pairs in DNA to amino acids is known. However, this only gives the linear sequencing of the amino acids in a protein. The three-dimensional structure of the protein must be determined in some other way. An essay would look at some aspect of the protein folding problem such as a quantum-mechanical calculation of molecules using a variant of the Hartree-Fock approximation, density functional methods, molecular dynamics and optimisation methods. A possible related topic is protein signalling.

Biological or quantum computing (Essay or Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

There has been recent progress in exotic forms of computing. Biological computing uses the properties of DNA (or possibly RNA) to perform massively parallel computations in a test-tube. The first computation experiment attempted in the laboratory was the travelling salesman problem. An essay would review the theory, algorithms and practical implementation issues of biological computing. A related topic is the theory, algorithms and a computer language which have been developed for quantum computing using the principles of quantum mechanics.

Analysis and modelling of the ionospheric dynamo, and the solar and lunar regular magnetic daily variations (Sq and L) (Project)
Supervisor: Dr D.J. Ivers (Carslaw room 623; phone 9351-3561)

The ionosphere is a thin conducting layer formed daily at about 106km altitude by ionisation due to solar radiation. Motion of the ionosphere caused primarily by solar heating of the atmosphere, but also by lunar (and solar) tides, in the main magnetic field of the Earth acts as a disturbance dynamo to produce variations in the total magnetic field. These variations in magnetic data from recent and current magnetic satellites - Oersted, SAC-C and CHAMP - and from magnetic observatories around the Earth via INTERMAGNET may be analysed to model the ionosphere, the solar and lunar regular magnetic daily variations. This project involves analysis and modelling using some of the available data.

Nonlinear integrable difference equations (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw room 522; phone 9351-4533)

The field of integrable difference equations is only about 10 years old, but has already caused great interest amongst physicists (in the theory of random matrices, string theory, or quantum gravity) and mathematicians (in the theory of orthogonal polynomials and soliton theory). For each integrable differential equation there are, in principle, an infinite number of discrete versions. An essay in this area would provide a critical survey of the many known difference versions of the classical Painlevé equations, comparisons between them, and analyse differing evidence for their integrability. Project topics would include the derivation of new evidence for integrability. The field is so new that many achievable calculations remain to be done: including derivations of exact solutions and transformations for the discrete Painlevé equations.

Exponential asymptotics (Project)
Supervisor: Prof N. Joshi (Carslaw room 522; phone 9351-4533)

Near an irregular singular point of a differential equation, the solutions usually have divergent series expansions. Although these can be “summed” in some way to make sense as approximations to the solutions, they do not provide a unique way of identifying a solution. There is a hidden free parameter which has an effect like the butterfly in chaos theory. This problem has been well studied for many classes of nonlinear ODEs but almost nothing is known for PDEs and not much more is known for difference equations. This project would include studies of a model PDE, like the famous Korteweg-de Vries equation near infinity, or a difference equation like the string equation that arises in 2D quantum gravity.

Mathematical Immunology (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw room 522; phone 9351-4533)

After almost three decades of knowledge about HIV/AIDS, it was only recently discovered that a cellular automata model is capable of replicating the long three-phase cycle in clinical data on T-cell populations. However, agreement with reality is still lacking. Asymptotics of cellular automata appear to be crucial to understanding this cycle. However, currently no such theory is known. This project can go in many directions. One direction is to consider different cellular-automata models for disease transmission. A second direction is to develop mathematical methods to work out limiting behaviours of cellular automata.

Phylogenetic algorithms in molecular biology (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Phylogenetic inference is the construction of the evolutionary history of a set of species (or taxa) by analysing the distances between their nucleotide sequences, protein structures or morphological characteristics. This problem is of fundamental practical importance in many areas of evolution and molecular biology. The methods that yield the best estimates of phylogenies are almost all based upon optimization problems that are NP-hard (cannot be solved in polynomial time). Thus many algorithms are heuristic methods that are not guaranteed to solve the problems exactly. Generally, a phylogenetic analysis may return hundreds of candidate evolutionary trees (if not thousands), determining the probable characteristics of the ”actual” tree is thus a difficult problem. This essay should begin with a summary of well established ”text-book” algorithms for inferring phylogenies that are mostly deterministic and then move on to a survey of current literature on non-deterministic algorithms such as genetic algorithms and visualisation algorithms for coping with the complexity of the solution space. (see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Artificial embryology in optimisation algorithms (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Many modern algorithms for optimising or modelling complex systems mimic naturally occurring phenomena such as evolution, neural networks, insect colonies and immune systems. Genetic algorithms and other evolutionary algorithms solve optimisation problems by describing a population of candidate solutions by sets of genes, and then applying mutation, recombination and selection operators to evolve the population and gradually obtaining superior solutions to the problem at hand. Until recently, most of these systems use the genes to parameterise or "describe" the solution: a "blueprint" approach. Real genes in biology do not describe the organism directly but instead determine a set of processes that interactively participate to build the organism: a "developmental" approach. Many practitioners in this field are calling this developmental approach "artificial embryology". An essay on this topic would explore recent developments in the use of evolutionary-developmental models for the optimisation of complex systems. (see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Level set theory for topological optimization (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Many interesting problems in physics and engineering require determining a shape or geometry that optimizes some physical process: for example, the size, shape and locations of holes in a special purpose microstructured optical fibre. There may be in addition geometric constraints: how close two surfaces can come, the maximum or minimum curvature allowed etc. A simple approach is to choose in advance some basic geometric building blocks: circles, wedges, ellipses and then optimize the numerical parameters of these objects. A much more interesting approach allows even the topology or connectedness (number of holes) to be varied. The geometry is chosen from the contours (or level sets) of some characteristic function. As this function is varied, the contours move around, and peaks, ridges and valleys can appear, merge or vanish is a smooth way. These ideas have already been successfully applied in other fields such as architecture.

Optimization of structural topology requires a combination of techniques to vary the level sets but also to recompute the solutions to the relevant differential equations (or other equations) using perturbation theory. Depending on the skill and interests of the student this project can go in a number of directions from a more computer science algorithms approach, to a computational science approach, or to studying perturbation theory of the differential equations themselves.
(see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Algebraic varieties in phylogenetics (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Phylogenetics is the reconstruction of the evolutionary tree relating species from the degree of similarity of their genetic sequences. Many different algorithms existing for inferring, approximating and searching for the correct tree topology and the correct lengths of the branches of the evolutionary tree.

Algebraic varieties are a branch of pure mathematics related to simultaneous polynomial equations. It turns out that certain algebraic varieties are invariant with respect to changes in the branch lengths but can be used as a test of the correct tree topology. This project will survey the field and also explore how these invariants can be used to enhance other algorithms used to search for the "best" evolutionary tree. A rare combination of pure and applied mathematics with important benefits in the emerging field of bioinformatics. (see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Representation in genetic algorithms (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Genetic algorithms are a popular technique used to search highly multidimensional spaces for the maxima or minima of complicated functions. The ability of a genetic algorithm to efficiently search the so called "fitness landscape" for both global and local optima can depend crucially on the representation of the search space itself. This project will look at optimization problems that can be simultaneously represented by different structures (trees, arrays, permutations, etc.) to explore how the choice of representation interacts with the genetic algorithm. This field is vast and this project could suit someone who wants to do an essay based project, but is also suitable for some original research. (see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Adaptive recombination in genetic algorithms (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

The recombination operator is an essential ingredient of any genetic algorithm allowing useful building blocks in the parent solutions to be combined into larger building blocks in the child solutions. The choice and design of the recombination operator is often the hardest part to get right, and an operator that is useful in the early stages of eveolution may be detrimental in later stages (the same operator that puts building blocks together, will also pull them apart...). In this project we look at changing the recombination operator with each generation. This involves developing a measure for the effectiveness of particular different reombination operators and also deciding how to change it. (see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Rugged Fitness Landscapes (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Kauffman developed a simple model for describing fitness functions that resemble rugged mountain ranges with multiple peaks, variation on different scales. This model, the NK model, consists of a total of N interacting parts which depend on at most K other parts of the system. Thus by varying the ratio of N and K different "levels" of complexity can be emulated. K=0 corresponds to models where independent causes produce independent effects, and K=N-1 corresponds to the absence of any correlations.

The NK model is a powerful tool to use as a test of the effectiveness of any particular proposed genetic algorithm. In this project I would like to explore how some basic genetic algorithms cope with NK fitness functions and which attributes of the genetic algorithm need to be changed as the value of K increases.
(see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Deceptive Imprinting in Immune Response (Essay or Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049)

Our immune systems have evolved a powerful memory mechanism. A naive immune system has to combinatorially search through many antibodies to find one that will bind with an invading virus. However, when presented with the same virus again the immune system can rapidly produce the original antibody. This response evolved as a powerful defense mechanism against viruses that do not change. Unfortunately, rapidly mutating viruses can fool such an immune system. A virus which is only slightly different will trigger the memory response rather than a new response and thus the body will incorrectly produce the older antibodies rather than search for newer and better ones. This type of deceptive imprinting is important in vaccine design and certain models of HIV infection. The project would suit a student who is comfortable with both Matlab and Mathematica, (or better still someone who can program in Python or C++). The essay/project can in several directions depending on the skill/interest of the student include coupled nonlinear ODEs, optimisation, or cellular automata.
(see also `http://www.maths.usyd.edu.au/u/leonp/NewStudentProjects.html`)

Bubbles, cells and ultrasound (Essay or Project)
Supervisor: Dr R. Thompson (Carslaw room 624; phone 9351-5782)

Ultrasound contrast agents are gas-filled microbubbles, of 1-20μm diameter, that can be injected into the body. They were introduced around 10 to 15 years ago to enhance existing techniques, but their behaviour has since been found to be extraordinarily complex, and the subject of much ongoing research. Understanding how introduced agents behave at a specific disease site within the body will open the door for new targeted techniques.

The behaviour of a microbubble depends on the applied ultrasonic field. The Rayleigh-Plesset model gives a nonlinear ODE for the time-dependent bubble radius. There are more elaborate versions of this model e.g. for contrast agents with shells, and a MATLAB package has been developed for microbubble simulations. This topic, with the focus on specific models and computer simulation, would suit a project. Alternatively an essay surveying ultrasound contrast agents, and the many open questions about their behaviour *in vivo*, could be undertaken by somone interested in the medical side.

Nonlinear Acoustics (Essay or Project)
Supervisors: Dr R. Thompson (Carslaw room 634; phone 9351-5782) and A/Prof C. Macaskill (Carslaw room 627; phone 9351-4163)

Acoustics relies heavily on linear or first order approximations. The linear theory works remarkably well for understanding a very wide range of phenomena - from musical sounds to noise control; from underwater scanning over kilometres to sub-millimetre measurements with biomedical ultrasound; from the behaviour of buildings and large man-made structures to communication and sensing in animals. This topic would begin with an overview of the nonlinear governing equations for finite amplitude disturbances in a fluid. Shock waves are formed when waves of finite amplitude are allowed to propagate over a sufficient distance in a lossless medium; dissipation tends to counteract the formation of shocks. Both these effects are captured in Burgers equation, and several related PDEs have been proposed to account for the finite size of sources and directional properties of beams. This topic could suit an essay or a project. A project would require some numerical skills.

Models for Biomedical Ultrasound (Essay or Project)
Supervisor: Dr R. Thompson (Carslaw room 624; phone 9351-5782)

Ultrasound has a well established role in medicine. The most well-known application is B-mode imaging, but there are many different applications including Doppler, power Doppler, intravascular, elastography and tissue harmonic imaging. Highly specialized measurements can be made of specific structures such as heart muscle, heart valves, bone, or atherosclerotic plaque. While there are mathematical and laboratory models for many aspects of these applications, it is surprising how much is not fully understood. Better understanding of ultrasound propagation and interaction with biological tissue is required, as well as technological developments. Essay or project topics in this area could be chosen to suit someone interested in biomedical engineering, medical imaging or biomechanics. Examples include: models for backscatter from blood, generation of tissue harmonics, measurement and calculation of medical ultrasound transducer fields *in vivo*, design of *in vitro* flow and imaging phantoms.

Relaxation oscillators, return maps and physiological rhythms (Essay or Project)
Supervisor: Dr M. Wechselberger (Carslaw room 628; phone 9351-3860)

Projects in my area of research are concerned with the study of oscillatory patterns of so called 'slow/fast systems'. These systems are ubiquitous in nature and control most of our physiological rhythms. E.g. one cycle of a heart beat consists of a long interval of quasi steady state interspersed by a very fast change of state, the beat itself. This very fast relaxation of energy leads to the notion of a relaxation oscillator and shows the appearance of multiple time-scales (slow/fast) in the system. To study periodic solutions in such systems one can analyse an associated return map. It is well known that already

1D maps can have a rich variety of dynamics, like periodic, quasiperiodic and chaotic solutions, the most famous example is the 1D circle map. Plotting the winding number versus a control parameter in this case yields the famous 'devil's staircase'.

Topics could range from a theoretical study of possible dynamics of return maps associated with relaxation oscillators to the analysis of a concrete physiological rhythm. For more information on possible topics, please see the research link on my webpage `http://www.maths.usyd.edu.au/u/wm/`

7 Rights and Responsibilities

Applied Mathematics 4 students will have access to the following.

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail and the World-Wide Web, as well as TEX and laser printing facilities for the preparation of essays and projects.
- A photocopying account paid by the School for assembling essay/project source material.
- After-hours access key to the Carslaw building. (A deposit is payable.)
- A pigeon-hole in room 728 — please inspect it regularly as lecturers often use it to hand out relevant material.
- Participation in the School's social events.
- Class representative at School meetings.

Applied Mathematics 4 students have the following obligations.

- Regular attendance at the regular weekly seminars in applied mathematics.
- Have regular meetings with project/essay supervisors, and meet all deadlines.
- Utilise all School resources in an ethical manner.
- Contribute towards the academic life in Applied Mathematics at the School of Mathematics and Statistics.

8 Scholarships, Prizes and Awards

The following prizes may be awarded to Applied Mathematics 4 students of sufficient merit. Students do not need to apply for these prizes, which are awarded automatically.

Joye Prize in Mathematics Value: **$5000, with medal and shield**
Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal
Awarded to Honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics 4 and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K.E. Bullen Memorial Prize Value: **$650**
Awarded annually on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics 4, provided that the student's work is of sufficient merit.

M.J. and M. Ashby Prize Value: **$250**
Offered annually for the best essay, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics 4, Applied Mathematics 4 or Mathematical Statistics 4.

Barker Prize Value: **$375**
Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

Norbert Quirk Prize No IV Value: **$130**
Awarded annually for the best essay on a given mathematical subject by a student enrolled in a Fourth Year course in mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of University Women (NSW) Prize in Mathematics
Value: **$100**
Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize Value: **$100**
For the best adjudged essay/project seminar presentation of an Applied Mathematics 4 student.

9 Life After Fourth Year

Postgraduate Studies:
Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the Director of Postgraduate Studies if interested in enrolling for a MSc or PhD at the School of Mathematics & Statistics. Students who do well in Applied Mathematics 4 may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers:
Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Programme, the Course Coordinator and the course lecturers may also provide advice and personal references for interested students.