Fourth Year Pure Mathematics

2019 Handbook
School of Mathematics and Statistics

THE UNIVERSITY OF
SYDNEY

http://www.maths.usyd.edu.au/u/UG/HM/

Contents

Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something completely different.

Goethe, *Maximen und Reflexionen*

Chapter 1

The Structure of Pure Mathematics Four

1.1 Introduction

In linguistics it is increasingly believed that universal features of language are reflections of the structure of the human brain and its perception of the world around us. In a similar fashion, mathematics is a universal language that has been developed to understand and describe how nature and life work. Mathematics, both in structure and development, is inextricably bound to our attempts to understand the world around us and our perceptions of that world. We see this in the mathematical descriptions and formulations of models in the theoretical and applied sciences: from physics, computer science and information theory on the one hand, to engineering, chemistry, operations research and economics on the other.

Just as remarkable is the way in which esoteric and abstract mathematics finds applications in the applied sciences. Indeed, one of the most exciting developments in science over the past decade has been the re-emergence of a dynamic interaction between pure mathematicians and applied scientists, which is bringing together several decades of the relatively abstract and separate development of pure mathematics and the sciences. Examples include the applications of singularity theory and group theory to symmetry-breaking and bifurcation in engineering; number theory to cryptography; category theory and combinatorics to theoretical and computational computer science; and, most spectacular of all, the developments of general field theories in mathematical physics based on the most profound work in complex analysis and algebraic geometry. Of course, this interaction is not one way. For example, there is the discovery of “exotic” differential structures on $\mathbb{R}^4$ utilising ideas from Yang-Mills theory.

There are many valid approaches to the study of Pure Mathematics in the final Honours Year. Thus, the course may be regarded as useful in its own right, or may lead on to an M.Sc. or Ph.D. or to a teaching position in University or High School. In another direction, what want a solid base from which to continue with studies in computer science or physics, for example. Finally, you may intend to seek employment with the CSIRO or in the operations research field, or in a financial institution. In the latter circumstances, one well-known advantage of studying mathematics is that mathematics gives training in a particular way of thinking and an analytic approach to problem solving. Mathematicians are highly adaptable (and employable).

The Fourth Year Honours program in Pure Mathematics caters for the various needs described above by offering a highly flexible and adaptable program, which is both interesting and challenging. We offer a combination of core courses, which introduce the major areas of mathematics, together with a smorgasbord of deeper courses that can be arranged to suit your personal requirements.

In brief, the Fourth Year course comprises the equivalent of six lecture courses, together with an essay project (counting as the equivalent of four lecture courses) and a 20 minute talk on the essay project.

A description of the various components of the course is given below. For detailed descriptions of the courses, the essay project, and so on, see the appropriate chapter in this Handbook.

1.2 The lecture courses

Students are required to be assessed on 6 units of approved lecture courses (or equivalent—see below).

In 2019 the lecture courses may be chosen from:

1. three PM4 core lecture courses, each worth 1 unit.
2. other PM4 lecture courses, each worth 1 unit. (These may presume some knowledge of one or more of the core lecture courses.)
3. third year *advanced* units of study, each worth 1 unit. (Students in Pure Mathematics 4 may take any 3(A) unit of study that they have not taken previously.)
4. Approved substitutions (up to the value of 2 units) by lecture courses given by other Departments. (See §1.5 below.)
5. Reading courses arranged with staff members (after consultation with the PM4 Coordinator).
6. Approved AMSI summer-school courses (after consultation with the PM4 Coordinator).
7. Approved AMSI grid room courses (after consultation with the PM4 Coordinator).

Read carefully the guidelines in §1.5 below.

Overall, the lecture courses offered at the level of PM4 and above are intended to introduce students to the major divisions of modern mathematics and provide a knowledge of some of the main ideas needed for the understanding of much of contemporary mathematics, while still reflecting the research interests within the pure mathematics research groups.

The "core" of Fourth Year is considered to include Commutative Algebra, Functional Analysis and Algebraic Topology. Students are strongly advised to take all of the core courses.

1.3 Pure Mathematics Honours/PG Lecture Courses for 2019

SEMESTER I

Algebraic Topology	Kevin Coulembier
Commutative Algebra	Ruibin Zhang
Functional Analysis	Alexander Fish
Representation Theory	Alex Molev

SEMESTER II

Riemannian Geometry with applications to Ricci Flow	Zhou Zhang
Introduction to quantum groups	Ruibin Zhang
Analytic number theory	Dzmitry Badziahin

If you are unsure about the combination of courses you should take, consult with your supervisor or the course coordinator. In any case, you are very welcome to attend all the lecture courses.

1.4 Pure Mathematics 3(A) Units of Study for 2019

SEMESTER I

Metric Spaces (MATH4061)
Rings, Fields and Galois Theory (MATH4062)
Nonlinear Ordinary Differential Equations with Applications (MATH4063)
Complex Analysis (MATH4079)

SEMESTER II

Differential Geometry (MATH4068)
Measure Theory and Fourier Analysis (MATH4069)

1.5 Tailor-made courses

Students may be permitted to take lecture courses of an essentially mathematical nature other than these, if the approval of the PM4 Coordinator is obtained. Possible examples include Applied Mathematics or Mathematical Statistics lecture courses, or lecture courses offered by other schools. Normally we require that at least four of the six course units be chosen from the Pure Mathematics lecture courses. Details of the Applied Mathematics

4 and Mathematical Statistics 4 options may be obtained from the coordinators (Applied Mathematics: Dr Robert Marangell, Carslaw 720, email am4coord@maths.usyd.edu.au, phone (02) 9351 5795 and Mathematical Statistics: A/Prof Uri Keich, Carslaw 821, email st4coord@maths.usyd.edu.au, phone (02) 9351 2307)

A number of staff are usually willing to supervise a reading course in their particular area of interest. Consult the course coordinator if you have a special topic in mind that might be acceptable as a reading course. Reading courses are generally a matter between the student and a willing member of the department, subject to the approval of the course coordinator. It is usually more work to take a reading course than a regular lecture class as no one is condensing the material into lectures for you. This option makes sense if there is a particular area you are interested in that is not covered by the lecture courses, but unless you have special reason for taking a reading course we advise taking the regular classes.

If you wish to do a **reading course** in Pure Mathematics, or substitute a course from outside Pure Mathematics, you should ask the lecturer to prepare a **summary** of the course and description of the **assessment**. This should then be submitted to the course coordinator for approval.

1.6 The essay project

The essay project (including the talk) counts as the equivalent of four PM4 units. Work on the essay project proceeds throughout the year and the finished essay is submitted near the end of the second semester. Note that it is also possible for the project to be supervised by a member of another department (or jointly).

1.7 The talk

As part of the essay project, students are required to give a talk about their project. The talk is worth 5% of the essay mark. The talk will usually take place in the week before the mid-semester break.

I fear that I bore you with these details, but I have to let you see my little difficulties, if you are to understand the situation.

Sherlock Holmes, *A Scandal in Bohemia*

Chapter 2

Entry, Administration and Assessment

2.1 Entry Requirements for Pure Mathematics 4

Students who have fulfilled the requirements of the faculty in which they are enrolled, and have satisfied the conditions below, are eligible to apply for Pure Mathematics 4. These conditions are that students should have:

(a) taken 24 credit points of third year mathematics units (see the senior pure and applied mathematics handbook) with at least 12 of these credit points in advanced level pure mathematics;

(b) obtained a credit average or better within advanced level third year mathematics subjects/a distinction average or better within normal level third year mathematics subjects.

Entry to PM4 is also subject to the approval of the Head of School and judgement of the Honours coordinator.

Note: Since we advise all PM4 students to take the core courses (Commutative Algebra, Algebraic Topology and Functional Analysis), the natural prerequisites for PM4 are Metric Spaces, Rings Fields and Galois Theory, and Measure Theory and Fourier Analysis. See the PM4 Coordinator for advice if necessary.

2.2 Actions to be taken

All students intending to take Pure Mathematics 4 should see the *PM4 Course Coordinator*, Prof Laurentiu Paunescu(Carslaw 721, phone (02) 9351 2969, email `pm4coord@maths.usyd.edu.au`) at their earliest opportunity, and in any case well before the beginning of the new teaching year. The Course Coordinator will advise you about choosing a supervisor and a topic for the essay project (see also §4.2 below).

2.3 Administrative arrangements

The PM4 Course Coordinator is in charge of Pure Mathematics 4 and should be consulted about any organisational problems that may arise.

In particular, students should note that the Course Coordinator's permission should be obtained if you wish to substitute courses from outside, or take a reading course or a postgraduate course. In the first instance, however, you should discuss such matters with your supervisor. Provided you can agree, the Course Coordinator's permission would then normally be a formality.

Please take particular note of the procedure to be followed if you are sick or other circumstances arise that may lead to late submission of your essay (see §4.4). Also note that at the end of first semester a progress report must be given to the Course Coordinator (see Chapter 5).

When we know that you are enrolled for PM4 you will be given a computer account. *The usual way in which messages for PM4 students will be distributed will be via e-mail.* Please remember to check your e-mail regularly.

2.4 Assessment

The possible results for fourth year are First Class Honours, Second Class Honours division 1, Second Class Honours Division 2, Third Class Honours and No Award (Fail), usually abbreviated I, II-1, II-2, III and F. The last two are rarely awarded.

Each PM4 lecture course is assessed at a time and in a manner arranged between the lecturer and the class. Usually, a written examination is held during the exam period immediately following the course; however, some courses are assessed entirely by assignment. It is undesirable to have examinations during term or to have many papers deferred to the end of the year. Each PM3 advanced course is assessed in the usual way. Students are informed about their performance as information becomes available.

Marks provided by the lecturers are scaled according to the lecturer's judgement as to what constitutes a I, II-1, II-2, III or F performance on each course. The marks from the best 6 units are counted towards the final mark. The essay is equivalent to 4 PM4 units; it accounts for 40% of the year's assessment.

As well as assessing the Fourth Year performance, the Department is required to make a recommendation for a grade of Honours based on the performance in all subjects over the four years. In exceptional cases, the grade of Honours awarded could differ from the level of performance in the Fourth Year.

2.5 Honours grades

The Faculty of Science has given the following guidelines for assessment of student performance in fourth year.

95–100 Outstanding First Class quality of clear Medal standard, demonstrating independent thought throughout, a flair for the subject, comprehensive knowledge of the subject area and a level of achievement similar to that expected by first rate academic journals. This mark reflects an exceptional achievement with a high degree of initiative and self-reliance, considerable student input into the direction of the study, and critical evaluation of the established work in the area.

90–94 Very high standard of work similar to above but overall performance is borderline for award of a Medal. Lower level of performance in certain categories or areas of study above.

Note: An honours mark of 90+ and a third year WAM of 80+ are necessary but not sufficient conditions for the award of the Medal. Examiners are referred to the Academic Board Guidelines on the award of Medals found in the general policy pages at the front of the Examiners' Manual.

80–89 Clear First Class quality, showing a command of the field both broad and deep, with the presentation of some novel insights. Student will have shown a solid foundation of conceptual thought and a breadth of factual knowledge of the discipline, clear familiarity with and ability to use central methodology and experimental practices of the discipline, and clear evidence of some independence of thought in the subject area. Some student input into the direction of the study or development of techniques, and critical discussion of the outcomes.

75–79 Second Class Honours, First Division – student will have shown a command of the theory and practice of the discipline. They will have demonstrated their ability to conduct work at an independent level and complete tasks in a timely manner, and have an adequate understanding of the background factual basis of the subject. Student shows some initiative but is more reliant on other people for ideas and techniques and project is dependent on supervisor's suggestions. Student is dedicated to work and capable of undertaking a higher degree.

70–74 Second Class Honours, Second Division – student is proficient in the theory and practice of their discipline but has not developed complete independence of thought, practical mastery or clarity of presentation. Student shows adequate but limited understanding of the topic and has largely followed the direction of the supervisor.

65–69 Third Class Honours – performance indicates that the student has successfully completed the work, but at a standard barely meeting honours criteria. The student's understanding of the topic is extremely limited and they have shown little or no independence of thought or performance.

The award of a medal is *not* made just on the basis of a numerical mark or formula. The merits of each eligible candidate are debated by the relevant Board of Examiners.

2.6 School Facilities

Pure Mathematics 4 students traditionally enjoy a number of privileges. These include:

- Office space and a desk in the Carslaw Building.

- A computer account with access to email and the internet, as well as TeX and laser printing facilities for the preparation of essays and projects.
- A photocopying account paid by the School for essay/project source material.
- After-hours access to the Carslaw Building. (A deposit is payable.)
- A pigeon-hole in room 728 – please inspect it regularly as lecturers often use it to hand out relevant material.
- Participation in the School's social events.
- Class representative at School meetings.

2.7 Scholarships, Prizes and Awards

The following scholarships and prizes may be awarded to Pure Mathematics 4 students of sufficient merit. (Note that unless the conditions of the prize state otherwise, as in the David G.A.Jackson Prize and the A.F.U.W. Prize, these prizes are also open to all Honours students in the School of Mathematics and Statistics.)

Joye Prize in Mathematics

To the most outstanding student completing fourth year honours in the School of Mathematics and Statistics.

Value: $5300 plus medal and shield.

George Allen Scholarship in Pure Mathematics

To a student proceeding to Honours in Pure Mathematics who has shown greatest proficiency in at least 24 credit points of Senior units of study in the School of Mathematics and Statistics.

Value: $1000.

Barker Prize

Awarded at the fourth (Honours) year examiner's meetings for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

Value: $550.

Ashby Prize

Offered annually for the best essay, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics 4, Applied Mathematics 4 or Mathematical Statistics 4.

Value: $360.

Norbert Quirk Prize No IV

Awarded annually for the best essay on a given mathematical subject by a student enrolled in a fourth year course in mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Value: $250.

David G. A. Jackson Prize

Awarded for creativity and originality in any undergraduate Pure Mathematics unit of study.

Value: $1100.

Australian Federation of Graduate Women: Prize in Mathematics

Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of B.A. or B.Sc. who graduates with first class Honours in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

Value: $175.

Rolf Adams Prize

This annual prize is awarded to the pure mathematics honours student who delivers the best talk.

Value: $100.

University Medal

Awarded to Honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Pure Mathematics 4 and a WAM of 80 or higher in 3rd year. More than one medal may be awarded in any year.

I dislike arguments of any kind. They are always vulgar, and often convincing.

Oscar Wilde, *The importance of being earnest*

Chapter 3

Course Descriptions

All courses offered in 2017 are 2 lecture per week courses and count as 1 unit. Some of the Semester II courses have one of the core courses as a prerequisite. In addition, any 3(A) courses, not previously examined, is available for credit. Each 3(A) course is run at 3 lectures and 1 tutorial per week, and counts as 1 unit. For substitutions by courses not given by Pure Mathematics see Section 1.3.

3.1 Fourth Year Courses — Semester I

Algebraic Topology

Lecturer: Kevin Coulombier

Algebraic Topology is certainly among the branches of pure mathematics undergoing the most rapid development in the last one hundred years or so. It has enormous influence on other major branches, such as algebra, algebraic geometry, analysis, differential geometry and number theory.

The typical problem of topology is to characterise or classify spaces. Some obvious intuitions, for example, Euclidean spaces of different dimensions are not the same, can be justified in a rigorous way using Algebraic Topology tools. In order to do this, we need to construct various invariants in algebraic terms. The homotopy groups and homology groups of a topological space are two important families of such invariants. The homotopy groups are easy to define but are in general very difficult to compute and the term in dimension one, called the fundamental group, serves as a reasonable example at honours level; the converse holds for the homology groups. Our goal for this course is to provide brief but complete discussion for both invariants, focusing on connecting the abstract side of the story with the geometric intuitions.

We begin with necessary preparations, introducing important notions such as homotopy and homotopy equivalence. Then the more intuitively defined fundamental group comes first. The homology theory for us features a few equivalent ways of defining the same algebraic objects. We begin with the most intuitive simplicial homology, then go for the most abstract singular homology, and finally reach cellular homology which is somewhat the most convenient. The properties can be summarized in the Eilenberg–Steenrod axioms, which is touched, together with the notions of category and functor, towards the end of the semester. Along the way, we include a lot of applications of these invariants, through degree theory for example.

Prerequisite: MATH3961: Metric Spaces

Assessment: There will be 2 assignments, together worth 20% of the assessment, and a final exam, worth 80%. The assignments and exam will mostly be based upon the suggested problems in the course notes.

Resources: The first few chapters of Allan Hatcher's book *Algebraic Topology* (Cambridge University Press, 2002) will be the guideline for us. (It is available online at http://www.math.cornell.edu/~hatcher.) We have other typed notes on the course web site (through school web site). Jonathan Hillman's notes are a good reference. The one by myself, serving as a more detailed plan for this semester, has the suggested problems in it, and we might have more problems added into the collection along the way.

Commutative Algebra

Lecturer: Ruibin Zhang

One of the most significant mathematical innovations of the 20th century was the development of "context-free geometry". The key idea of this is that the study of the simultaneous solutions of polynomial equations such as $x^4 + y^4 + z^4 - 5x^2y^2 = 0$ may be carried out in a way that is independent of the domain in which the variables $x, y, z, \ldots$ lie. For example they may lie in $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}$ or $\mathbb{F}_q$, and prior to these developments the study of solutions in those domains would have been regarded as separate disciplines. Thus new common ground now exists between geometry over various domains. There is also now a much better understanding of much studied concepts such as the "multiplicity" of a higher order intersection of curves or their higher dimensional analogues. The foundations for these spectacular advances were laid largely by the Paris school of Grothendieck in the 1950's and 60's building on the work of many mathematicians over many centuries, but most importantly on that of Hilbert in the 1920's. Its basis is the abstraction of geometry by algebra, and this course, on commutative algebra, is intended to be an introduction to the basic ideas of the subject.

The course will cover the basic concepts of commutative algebra, and illustrate them by giving geometric interpretations as far as possible.

Course content: The course will include topics from the following.

i) Basic ideas. Commutative algebras over a field; affine varieties and commutative algebras; examples. Noetherian rings, Hilbert basis theorem; ideals, prime and primary ideals, decomposition theory. Localisation.

ii) Integral extensions, the Nullstellensatz, geometric consequences. Definition of Spec(R). Noether normalisation. Filtered and graded rings; completions; flatness. Homological functors Ext and Tor.

iii) Dimension theory; Poincaré or Hilbert series; morphisms of varieties and their fibres.

iv) Tangent and cotangent spaces; local properties of morphisms. Examples: deteminantal varieties, group schemes.

Prerequisite: Students are expected to have a thorough knowledge of linear algebra, and to have taken advanced algebra courses in third year.

Assessment: Two assignments worth 20% each, and a written examination worth 60% to be held at the end of Semester 1.

Resources:

[1] M. F. Atiyah and I. G. Macdonald, *Introduction to commutative algebra*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969. MR0242802

[2] Nicolas Bourbaki, *Commutative algebra. Chapters 1–7*, Elements of Mathematics (Berlin), Springer-Verlag, Berlin, 1998. Translated from the French; Reprint of the 1989 English translation. MR1727221

[3] David Eisenbud, *Commutative algebra*, Graduate Texts in Mathematics, vol. 150, Springer-Verlag, New York, 1995. With a view toward algebraic geometry. MR1322960

[4] Hideyuki Matsumura, *Commutative ring theory*, Cambridge Studies in Advanced Mathematics, vol. 8, Cambridge University Press, Cambridge, 1986. Translated from the Japanese by M. Reid. MR879273

[5] Serge Lang, *Algebra*, 3rd ed., Graduate Texts in Mathematics, vol. 211, Springer-Verlag, New York, 2002. (This is a reference for background in algebra.) MR1878556

Functional Analysis

Lecturer: Alexander Fish

Modern functional analysis is the study of infinite dimensional vector spaces and linear transformations between such spaces. Thus it can be thought of as linear algebra in an infinite dimensional setting.

To get the theory going we add some topology to the vector space: A *normed vector space* is a vector space with a concept of distance, and a complete normed vector space is called a *Banach space*. If the norm of the Banach space comes from an inner product (like the dot product in $\mathbb{R}^n$) then the Banach space is called a *Hilbert space*. These spaces should be thought of as the direct generalisation of $\mathbb{R}^n$ into the infinite dimensional setting, because they share many geometric properties in common with the former spaces.

We will begin by studying the geometric and structural properties of Banach and Hilbert spaces, illustrating the theory with a wide range of examples. We prove the *Stone–Weierstrass Theorem*, which has applications to the approximation of continuous functions on compact sets by polynomials and trigonometric polynomials.

We continue with a study of linear operators on Banach and Hilbert spaces, and prove three fundamental theorems of functional analysis: The *Open Mapping Theorem*, the *Principle of Uniform Boundedness*, and the *Hahn-Banach Theorem*. We will illustrate the importance of these theorems by listing many immediate corollaries.

We will also consider *spectral theory* for operators between Banach and Hilbert spaces. This is the generalisation of the notion of eigenvalues and eigenvectors, and proves to be extremely important in many applications, such as the modern theory of partial differential equations, mathematical physics, and probability theory.

Assessment: Assignments, Quizzes, and an final Exam.

Prerequisite: MATH3961 Metric Spaces (Adv) and MATH3969 Measure Theory and Fourier Analysis (Adv)

Resources:

[1] John B. Conway, *A course in functional analysis*, 2nd ed., Graduate Texts in Mathematics, vol. 96, Springer-Verlag, New York, 1990. MR1070713

[2] Kôsaku Yosida, *Functional analysis*, 6th ed., Grundlehren der Mathematischen Wissenschaften, vol. 123, Springer-Verlag, Berlin-New York, 1980. MR617913

[3] Daniel Daners, *Introduction to Functional analysis*, Lecture Notes, University of Sydney, 2018.

Representation Theory

Lecturer: Alex Molev

Representation theory is a major area of algebra with applications throughout mathematics and physics. Viewed from one angle, it is the study of solutions to equations in non-commuting variables; from another angle, it is the study of linear algebra in the presence of symmetry; from a third angle, it is the study of the most tractable parts of category theory. Historically, the representation theory of finite groups was developed first, and the many applications and beautiful special features of that theory continue to recommend it as a starting point. However, it is important to appreciate the underlying principles which unify the representation theory of finite groups, Lie algebras, quivers and many other algebraic structures.

Outline:

Basic notions: modules over associative algebras, submodules and quotients, direct sums, irreducible and indecomposable objects, Schur's lemma.

General results: characterisations of semisimplicity (complete reducibility), density theorem and absolute irreducibility, Jordan-Hölder theorem, Krull-Schmidt theorem, Wedderburn-Artin theorem.

Representations of finite groups: Maschke's theorem, characters, orthogonality, character tables, induced representations, Frobenius-Schur indicators, modular representations.

Representations of other algebraic structures: representations of the Lie algebra sl_2, Gabriel's theorem on representations of quivers.

Assessment: Two assignments worth 20% each, and a final examination worth 60% at the end of semester.

Resources:

[1] Pavel Etingof, Oleg Golberg, Sebastian Hensel, Tiankai Liu, Alex Schwendner, Dmitry Vaintrob, and Elena Yudovina, *Introduction to representation theory*, Student Mathematical Library, vol. 59, American Mathematical Society, Providence, RI, 2011. (Library call number 512.46/6). MR2808160

[2] Gordon James and Martin Liebeck, *Representations and characters of groups*, Cambridge Mathematical Textbooks, Cambridge University Press, Cambridge, 1993. (Library call number 512.2/51). MR1237401

[3] Steven H. Weintraub, *Representation theory of finite groups: algebra and arithmetic*, Graduate Studies in Mathematics, vol. 59, American Mathematical Society, Providence, RI, 2003. (Library call number 512.2/72). MR1984740

3.2 Fourth Year Courses — Semester II

Riemannian Geometry and Applications to Ricci Flow

Lecturer: Zhou Zhang

Differential Geometry and Riemannian Geometry are two major branches of pure mathematics. By name, they focus respectively on the smooth (i.e. differentiable) and metric (i.e. Riemannian) structures of the underlying objects, manifolds (for this course) which are the generalisation of more classic objects, curves and surfaces. The interactions between them in the last fifty years or so have triggered some of the most exciting developments in mathematics, for example, the pivot role of Ricci flow in settling Poincaré Conjecture and Geometrisation Program. In principle, their strength lies in the investigation on the deep relation between geometry and topology of mathematical objects.

From my personal point of view, although modern Differential Geometry and Riemannian Geometry can be viewed as the natural extension of more classic geometry, there are still major differences in between. Classic geometry was more of the collection of many seemingly isolated topics, sometimes featuring miracles through substantial computations, while the modern study, pioneered by mathematical giants including Gauss, Riemann, Cartan, Weyl and Chern, has put more emphasis on structure of general theory and scheme for complicated calculations.

For this honours course, we plan to explore an original way in introducing these abstract definitions by relating them to more intuitive notions touched before, in senior units for example. The following is a rough list of content.

i) Preparation: surface in Euclidean space, fundamental forms and Gauss-Bonnet Theorem.

ii) Manifold: tangent, cotangent and tensor bundles.

iii) Riemannian manifold: metric, Levi-Civita connection, curvatures; geodesic, Jacobi field, Bonnet-Myers Theorem and space forms.

iv) Flow basics: heat flow, Ricci flow and mean curvature flow.

Assessment: There will be two assignments worth 20% each. The final exam counts 60% of the total assessment.

Prerequisite: MATH3961 Metric Spaces. MATH3968 Differential Geometry is recommended but not required.

Resources: (not required)

[1] Manfredo Perdigão do Carmo, *Riemannian geometry*, Mathematics: Theory & Applications, Birkhäuser Boston, Inc., Boston, MA, 1992. MR1138207

[2] Sylvestre Gallot, Dominique Hulin, and Jacques Lafontaine, *Riemannian geometry*, 3rd ed., Universitext, Springer-Verlag, Berlin, 2004. MR2088027

[3] John M. Lee, *Riemannian manifolds*, Graduate Texts in Mathematics, vol. 176, Springer-Verlag, New York, 1997. An introduction to curvature. MR1468735

[4] Frank Warner, *Lectures on Differential Geometry*, Chelsea Pub. Co., 1982.

Analytic Number Theory

Lecturer: D. Badziahin

The aim of this course is to study various analytical methods for tackling problems in number theory. Famous examples include Prime Number Theorem about the asymptotic density of primes and Dirichlet Theorem about prime numbers in arithmetic progressions. We will see why zeroes of the Riemann zeta function are so important that one million dollars problem is related with them.

Another area which will be covered by this course is Diophantine approximation. It investigates various approximational properties of real numbers, points in $\mathbb{R}^n$ or, more generally, of elements in metric spaces. We will see that different real numbers can be approximated by rationals with different efficiency. The sets of numbers or points sharing the same approximational properties usually have very complicated fractal structure. We will see how to compute their "size" in terms of Lebesgue measure.

The approximate list of covered topics:

Distribution of primes

- Euler's proof of infinitude of prime numbers;
- Riemann zeta function and its continuation to the complex plane;
- Prime Number theorem;
- Dirichlet characters and L-functions.
- Dirichlet's proof of infinitude of prime numbers in arithmetic progressions;

Diophantine approximation

- Approximation of irrational numbers by rationals, Dirichlet theorem.
- Continued fractions and best approximants;
- Minkowski's theorem about lattice points in $\mathbb{R}^n$ and the generalisation of Dirichlet's theorem to multidimensional spaces.
- Khintchine theorem about about the approximational behavior of almost all real numbers.

Assessment: Assignments, worth 20% each and a final exam worth 60% at the end of semester.

Prerequisite: Students are expected to be familiar with complex analysis and have basic understanding of Lebesgue measure. Number Theory and Cryptography course will be helpful but is not required.

Resources:

[1] G. H. Hardy and E. M. Wright, *An introduction to the theory of numbers*, 5th ed., The Clarendon Press, Oxford University Press, New York, 1979. MR568909

[2] Harold Davenport, *Multiplicative number theory*, 3rd ed., Graduate Texts in Mathematics, vol. 74, Springer-Verlag, New York, 2000. Revised and with a preface by Hugh L. Montgomery. MR1790423

[3] Wolfgang M. Schmidt, *Diophantine approximation*, Lecture Notes in Mathematics, vol. 785, Springer, Berlin, 1980. MR568710

Introduction to quantum groups

Lecturer: R. Zhang

Quantum groups are a class of Hopf algebras discovered by Drinfeld and Jimbo in the study of exactly soluble models in statistical mechanics in the middle of 80s. They are "quantisations" of universal enveloping algebras of semi-simple Lie algebras and Kac-Moody Lie algebras in some appropriate sense (hence their name). Quantum groups have applications in many areas of mathematics and physics, and had profound impact on representation theory, low dimensional topology, the theory of Yang-Baxter type exactly soluble models and conformal field theory, among other subjects.

This course is an introduction to the theory of quantum groups associated with semi-simple Lie algebras at generic q. We will start by explaining the basics of Lie algebras needed in the course, then move on to studying quantum groups. Thus no prior knowledge of Lie algebras will be assumed.

The following material will be covered.

Basics on Lie algebras The general linear and special linear Lie algebras as examples; structure of semi-simple Lie algebras; finite root systems, Serre's theorem; finite dimensional representations of the general linear Lie algebra, tensor representations

The quantum group $\mathrm{U}_q(\mathfrak{sl}_2)$ Definition of $\mathrm{U}_q(\mathfrak{sl}_2)$, Hopf algebra structure, finite dimensional representations; universal R-matrix, braid group representations; the Jones polynomial of knots

Quantum groups of simple Lie algebras Definition of $\mathrm{U}_q(\mathfrak{g})$, Hopf algebra structure, finite dimensional representations, representation theory of $\mathrm{U}_q(\mathfrak{sl}_n)$ in detail

The universal R**-matrix and the centre** Non-degenerate pairing between opposite quantum Borel subalgebras, quantum double construction, universal R-matrix, the centre, Harish-Chandra theorem

Bases of quantum groups Braid group action on $\mathrm{U}_q(\mathfrak{g})$, PBW basis, canonical basis

Assessment: Two assignments worth 20% each, and a written examination worth 60% to be held at the end of the semester.

Resources:

[1] Vyjayanthi Chari and Andrew Pressley, *A guide to quantum groups*, Cambridge University Press, Cambridge, 1994. MR1300632

[2] James E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer-Verlag, New York-Berlin, 1972. Graduate Texts in Mathematics, Vol. 9. MR0323842

[3] Jens Carsten Jantzen, *Lectures on quantum groups*, Graduate Studies in Mathematics, vol. 6, American Mathematical Society, Providence, RI, 1996. MR1359532

[4] George Lusztig, *Introduction to quantum groups*, Progress in Mathematics, vol. 110, Birkhäuser Boston, Inc., Boston, MA, 1993. MR1227098

A man ought to read just as the inclination leads him; for what he reads as a task will do him little good.

Samuel Johnson

Chapter 4

The Essay

4.1 Introduction

q The essay project has several objectives. First and foremost, it is intended to provide an essentially open-ended framework whereby you may pursue, develop and discover your interests in mathematics unencumbered by syllabus and the prospect of eventual written examination. Basic to this process is the use of the library and communication with others, most especially your supervisor. The writing of the essay is a most valuable part of the project. The very act of writing is an invaluable aid to comprehension. A good essay should be carefully organised, clear, readable by others, laid out well, properly referenced and convey the essential ideas. Attainment of such writing skills is of great benefit whether or not you elect to stay in mathematics.

One point should, perhaps, be emphasized: the essay project is *not* generally intended to be a contribution to original research; however, the essay must clearly demonstrate that you understand and have mastered the material. Originality in presentation or view in the essay is required.

4.2 Choosing a Supervisor and topic

Choosing a supervisor and topic are the first two things that you should do, and are really not two choices, but one. It is recommended that you begin in the long vacation (preceding your fourth year) by seeking out members of staff and asking them about their interests and topics they would be keen on supervising. (See also Chapter 5 below.) It is a good idea to ask them about their particular method of supervising and other questions important to you. Do not feel you must settle for the first person you talk to!

All staff members, lecturer and above, are potential supervisors.

There is not necessarily any correlation between supervising style and lecturing style. Also, the subject a lecturer taught you may not be their real area of interest. You should try to decide on a supervisor and topic *before* the start of first semester. Most staff members will be available during the last two weeks of the long vacation; if you have not arranged a topic and supervisor at the beginning of the long vacation, you will probably have to organise your supervisor and topic during these last two weeks.

Changes in supervisor and/or topic are possible during the year (the earlier the better!). If you do change supervisors then you must notify the PM4 Coordinator.

It is a good idea to have a provisional topic and supervisor in mind at the beginning of the long vacation. Your potential supervisor can then suggest some reading for you to do over the long vacation, and, if you have second thoughts about the topic or supervisor, it is then easy to change before the first semester starts.

4.3 Essay content and format

The essay must start with an introduction describing the objective and contents of the essay. The essay may end with a summary or conclusion; however, this is optional. Should you wish to make any acknowledgements, they should appear on a separate page, following the introduction.

You should aim at the best scholarly standards in providing bibliographic references. In particular, clear references to cited works should be made, where appropriate, throughout the text. Furthermore, it is not acceptable to base large portions of your essay on the existing literature and whenever part of your essay closely follows one of your sources this must be *explicitly acknowledged* in the text. References should not appear in the bibliography unless they are referred to in the text. For the format of the references see the appendix.

The essay should be clear, coherent, self contained and something that others (your fellow students and other non-specialists in the topic) can read with profit. The essay should not exceed (the equivalent of) 60 pages one and a half spaced type of normal TeX font size (that is, as on this page). About 40 to 50 pages would normally be acceptable. Students are asked to try to keep their essays within these limits; overly long essay may be penalized. Supervisors should advise their students accordingly.

Take pains over style, especially clarity, precision and grammar. Aim at readability for the non-specialist. Avoid starting sentences with symbols. Aim for succinct statements of theorems and lemmas. Break up long proofs into lemmas. Cross reference previous results and notation, as this markedly improves readability.

Finally, the essay must be typed or printed and prepared in accordance with the instructions listed in the appendix. You should prepare your essay using a word-processing program such as LaTeX.

4.4 Submission of essay, assessment, corrections

Three bound copies of the essay should be given to the PM4 Course Coordinator for marking no later than noon on Monday of Week 13 in Semester 2 or Semester 1 in case you finish mid year. A PDF of the essay should also be submitted to the PM4 Course Coordinator by email by this deadline.

Any students submitting their essays past this time can anticipate a penalty of up to 5% per day or part thereof. Essays which are more than one day late may not be accepted. If, during the year, illness or other personal circumstances give a genuine reason for late submission of the essay, such matters should be reported to the Course Coordinator and your supervisor. Such circumstances should be reported as soon as possible, not at the last minute!

Each essay will be read independently by at least two members of the School. (The number of readers will depend on the staff available.) The candidate's supervisor may or may not be one of the readers. The markers may suggest corrections that should be made to the manuscript. If corrections are required, a final corrected copy of the essay should be given to the Course Coordinator for School records. If no corrections are required, one of the markers' copies will normally be kept by the School and the remaining two copies returned to the candidate.

4.5 Time management and progress reports

At the end of the first semester you should write a summary (approximately one page in length) of your essay project and progress and give this to the Course Coordinator. This should include a description of the project, the progress made in Semester 1, and what will be achieved in Semester 2. This must be approved by your supervisor before submission to the Course Coordinator.

Here are some rough guidelines and deadlines:

- Select supervisor and topic – before the beginning of first semester
- Reading, discussion and understanding – first semester
- Start work on first draft – by the end of first semester
- Final proofreading – mid-semester break

Do not underestimate the time it takes you to do the actual writing. Often it is not until you start writing that you will settle on a final view, or realise that you have misunderstood a particular part of the theory. Allow yourself sufficient time both for the typing and proof reading of the manuscript.

It is strongly advised that you provide your supervisor with drafts of your essay as soon as possible so that he/she may provide constructive feedback. In any case a complete draft must be submitted to your supervisor by 2 weeks before the final essay is due, namely Monday of Week 11.

The essay should be submitted by Monday of Week 13.

4.6 Your supervisor

To get the most benefit from the course, you should work closely with your supervisor. To this end, you may set up a regular hour each week to meet and discuss progress and problems with your essay project. Alternatively, you might come to some more informal arrangement.

You can expect your supervisor to:

- Help you select or modify your topic;
- Direct you to useful sources on your topic;
- Explain difficult points;
- Provide feedback on whether you are going in the right direction;
- Advise you on other course matters.

The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact.

A. Whitehead, *Science and the Modern World*

Chapter 5

Sample Essay Topics

Here are some topics or areas of interest suggested by members of the Department for 2019. Please note that this list is *not* intended to be complete, the topics suggested are perhaps best regarded as a guide to the likely interests of the proposer, and other staff members are willing to act as essay supervisors. The topics are grouped according to the Research Group to which each staff member belongs.

5.1 Algebra Research Group

Dr Nathan Brownlowe – Carslaw 635

Operator Algebra is the study of continuous linear operators on topological vector spaces. A large class of examples comes from C^*-algebras, which are closed, self-adjoint collections of bounded linear operators on Hilbert space. I study the structure of C^*-algebras associated to a number of mathematical objects, including algebraic objects like semigroups, groups, and groupoids; combinatorial objects like graphs, and their higher-dimensional analogues; and other mathematical objects, including graphs of groups, and Hilbert bimodules. I am happy to supervise projects in any of these areas.

Dr Kevin Coulembier – Carslaw 632

I would like to suggest two topics, as attached below.

i) Algebraic K-theory

   K-theory is a branch in mathematics connected to geometry, topology, representation theory, physics and number theory. K-theory assigns invariants, called K-groups (K_i for i a non-negative integer), to certain geometric and algebraic objects. These K-groups contain a lot of information about the object, but are difficult to compute. Even the K-theory of the ring of integers is unknown!

   One possible project here aims at understanding K_1. Historically, the first definition of K_1 of a ring is due to Bass and Schanuel (using the general linear group). Quillen defined K-groups for a ring using the 'plus'-construction. Later he improved this definition by introducing the 'Q'-construction for any exact category (using simplicial sets). A possible project is to understand some of these constructions, show they yield the same abelian group K_1, and/or compute K_1 in some explicit examples.

ii) Galois correspondences

In Galois theory one has a correspondence between intermediate field extensions and the Galois group of a field extension. In algebraic topology (PMH1) one has a correspondence between the fundamental group of a topological space and the covering spaces of said space. There is an ostensible analogy between both, which led to the former correspondence also being referred to as a Galois correspondence.

The book 'Galois Groups and Fundamental Groups' by Szamuely digs a bit deeper to explain this analogy by making connections between the fields. We could explore (some of) these connections between Galois theory, algebraic topology, differential equations and algebraic geometry.

A/Prof David Easdown – Carslaw 619

A/Prof David Easdown will be on study leave in Semester 2 of 2019 so not available for supervision.

Professor Anthony Henderson – Carslaw 717

I can supervise Honours projects in various areas of representation theory and algebraic geometry. Some specific suggestions are:

i) *ADE classifications.* A remarkable number of different classification problems (finite groups of orientation-preserving symmetries of complex 2-dimensional space, simple Lie algebras over an algebraically closed field, quivers of finite representation type, normal isolated surface singularities, etc.) have essentially the same answer, given by the so-called ADE graphs. A good essay project would be to explain the solutions to two or more of these problems and why they are connected.

ii) *Nilpotent orbit closures.* A square matrix is nilpotent if its sole eigenvalue is 0; this turns out to be equivalent to requiring that its similarity class has the zero matrix in its closure (for a suitable topology). The latter definition of nilpotency applies to elements of any group representation. The topology of nilpotent orbit closures has important connections to various parts of algebraic combinatorics and representation theory, and continues to be an active area of research.

iii) *Characters of finite groups of Lie type.* According to the classification of finite simple groups, most of them are groups of Lie type, such as the projective special linear group over a finite field. Thanks to the work of Lusztig (among others), there are algorithms to compute the irreducible characters of these groups, which generalize the more explicit formulas given by Green in the case of the finite general linear group. Explaining even a small part of this theory would make a substantial essay.

Professor Gus Lehrer – Carslaw 813

There are several possible topics, which come from two basic themes.

i) The first theme is that of *symmetries of algebraic varieties*; the solution sets of polynomial equations often have symmetries (for example when the defining polynomials are permuted by a group). This leads to actions on the homology of interesting topological spaces, such as discriminant varieties and toric varieties, the latter being defined by monomials associated with polyhedral cones. Topics available here include: computing actions via rational points; geometry of certain classical algebraic varieties; topological and geometric problems arising from reflection groups.

ii) The second theme is the *representation theory of associative algebras*. There are many semisimple algebras (with easily described representations) which may be deformed into non-semisimple ones by variation of parameters. These occur in algebra (Brauer, Hecke algebras), in mathematical physics (Temperley-Lieb algebras) and topology (BMW algebras, and such like). "Cellular theory" allows one to reduce deep questions about these deformations to (usually hard) problems in linear algebra. There are several possibilities for essay topics in this area.

Professor Andrew Mathas – Carslaw 718

I would be happy to supervise a fourth year essay on any topic in representations theory, or combinatorics. My main research interests are the representation theory of the symmetric groups and related algebras (such as Hecke algebra, Ariki Koike algebras, Schur algebras, general linear groups, Brauer algebras, Solomon's descent algebras, . . .), with an emphasis of the non-semisimple case—which is where things start to get interesting, and more difficult!

Possible topics include:

i) *The modular representation theory of finite groups.* In characteristic zero every representation of a finite group can be decomposed, in a unique way, as a direct sum of irreducible representations. For fields of positive characteristic this is no longer the case, but nevertheless the number of times that a given irreducible module can arise as a composition factor of a representation is uniquely determined. Possible projects in this area range from classifying the number of irreducible representations of a finite group, to studying the Brauer and Green correspondences.

ii) *Representations of symmetric groups.* The representation theory of the symmetric group is a rich and beautiful subject that involves a lot of algebra and combinatorics. Possible projects here include character formulae, classifying homomorphisms, computing decomposition matrices, Murphy operators, the Jantzen sum formula, ...

iii) *Brauer algebras.* The Brauer algebras arise naturally from the representation theory of the symplectic and orthogonal groups, but they can also be understood from a purely combinatorial viewpoint in terms of a "diagram calculus". Possible topics include character formulae, classifying semisimplicity, branching theorems, ...

iv) *Seminormal forms.* For many algebras it is possible to give "nice" generating matrices for the irreducible representations in the semisimple case. These explicit matrix representations are called seminormal forms. The study of the seminormal forms, and the resulting character formulae, for one or more algebras would make an interesting essay topic.

Professor Alex Molev – Carslaw 707

i) *Symmetric functions* The theory of symmetric functions is a classical area of algebraic combinatorics. It is closely related to geometry of algebraic varieties, representation theory of finite groups and Lie algebras, and has a number of applications in mathematical physics. In this project we will study multiparameter symmetric functions from the combinatorial and algebraic viewpoints.

ii) *Lie algebras and quantum groups.* The study of quantum groups has occupied a central stage in mathematics research for the past two decades. The groundwork for this field was laid in the mid-80s. 'Quantum groups' refer to a range of Hopf algebras that are deformations (quantizations) of either algebras of functions on groups, or universal enveloping algebras. The aim of the project is to study families of quantum groups associated with the classical Lie algebras.

Dr Oded Yacobi – Carslaw 724

My broad interests are in representation theory, especially in problems of geometric and combinatorial flavour. Most recently, I?ve been interested in categorical representation theory, which is the study of actions of groups or Lie algebras on categories rather than vector spaces. This is a very new subject, with amazing applications and much yet to be discovered. If you are interested in representation theory and/or category theory, then please feel free to contact me about possible Honours topics.

Professor Ruibin Zhang – Carslaw 722

I can supervise Honours projects in various areas of Lie theory and mathematical physics. Some possible topics are:

i) *Quantum groups.* Quantum groups are 'quantized versions' of universal enveloping algebras of Lie algebras. They originated from the study of the Yang-Baxter equation in physics in the 1980s, and have had very significant impact on many branches of mathematics and physics in recent years. Research on quantum groups is very active, and the subject is rapidly developing.

ii) *Infinite dimensional Lie algebras.* Infinite dimensional Lie algebras play a key role in conformal field theory and the theory of strings. Typically the Hilbert space of a physical system forms a positive energy module over the Virasoro or a Kac-Moody algebra, where the energy operator is some special element of the algebra. The study of the states and the energy spectrum of the physical system thus may be treated algebraically within the representation theory of these infinite dimensional Lie algebras. Thesis topics in this area involve studying such representations of the Virasoro and Kac-Moody algebras that are most commonly used in physics.

iii) *Lie superalgebras and supersymmetry.* Supersymmetry is a basic principle that ensures that the fundamental laws of physics are the same for bosons and for fermions. It has permeated many areas of pure mathematics in recent years, leading to deep results such as the Seiberg-Witten theory and mirror symmetry. The algebraic perspective of supersymmetry is the theory of Lie superalgebras, which came to existence in the late 1970s, and is still actively studied today. We shall develop the structure and representations of Lie superalgebras such as the general linear superalgebra and its subalgebras.

iv) *Quantum field theory and gravity.* Quantum field theory is the conceptual framework for formulating fundamental laws of physics and answering questions about the structure of the physical world. It provides the mathematical means for studying quantum systems of infinitely many degrees of freedom, and making definite predictions that can be tested at experimental facilities like the LHC at CERN. A very active area is the development of a quantum theory of gravity, which is necessary for understanding the structure of spacetime at the Planck scale. Thesis topics in this area involve the study of proposals of quantum theories of gravity using noncommutative generalisations of Riemannian geometry.

5.2 Computational Algebra Research Group

Professor John Cannon – Carslaw 618

i) *Computational Number Theory.* For example:

- Primality testing and factorization,
- Constructive algebraic number theory,
- Computation of Galois groups.

ii) *Computational Group Theory.* For example:

- Algorithmic methods for finitely presented groups,
- Algorithmic methods for permutation groups,
- Computational representation theory,
- Constructive invariant theory.

iii) *Computational Differential Algebra.* For example:

- The Risch algorithm for indefinite integration.

5.3 Geometry, Topology and Analysis Research Group

A/Prof Dmitry Badziahin — Carslaw 634

I do research in the area of Diophantine approximation, a branch of analytic number theory. At its core, it investigates, how well are real numbers (or some other objects) approximated by rationals. Possible topics for essays include:

- *Winning sets and their applications to Diophantine approximation.* Winning sets were firstly invented by Schmidt in 1960's. They have a surprising property: despite of being very small (basically, of zero Lebesgue measure), any countable intersection of winning sets is still winning and hence is uncountable. It appears that many sets of numbers or points in $\mathbb{R}^n$ sharing certain approximational properties are winning. Projects on this topic will involve the study of the classical Schmidt winning sets and their analogues. Then it will end up with checking a winning property for some set from the area of Diophantine approximation.

- *Approximational properties of Mahler numbers.* Within this topic we will start with the study of continued fractions in the space of Laurent series. Then we will move to Mahler functions — the class of functions which satisfy certain functional equations. We will see that in many cases their continued fraction can be computed by some recurrent formulae. Finally we will investigate, based on that information, how well can values of Mahler functions be approcimated by rational numbers.

- *Modern factorisation algorithms.* This topic stays a bit outside of my current research but it may be interesting for some students. We will consider one of the fastest modern factorisation algorithms: elliptic curve method or number field sieve. We investigate their computational complexity, their strengths and weaknesses.

Dr Emma Carberry – Carslaw 723

My primary research areas are differential geometry and integrable systems, although I also use methods from complex algebraic geometry in my work. I have listed some specific areas below in which I would be happy to supervise an essay but this list is far from exhaustive; if you have other geometric interests please feel free to contact me for further ideas.

i) *Curves and their Jacobians.* Algebraic curves (smooth algebraic curves are also called compact Riemann surfaces) and line bundles on them are utilised in many areas of mathematics. A project here could go in many directions, a good basic reference is Philip Griffiths' book "An Introduction to Algebraic Curves".

ii) *Spectral Curves and Integrable Systems.* There is an important class of differential equations that can be written in a particularly simple form, called Lax form. For example, the equations describing a minimal surface in a compact Lie group or symmetric space can be written in this form, as can equations governing Higgs bundles. It is a beautiful fact that solutions to such differential equations on the complex plane (satisfying a finiteness condition) are in one-to-one correspondence with purely algebro-geometric data, consisting of an algebraic curve and a line bundle. The curve is called a spectral curve, and this correspondence gives one powerful new tools with which to attack the original geometric problem. This project is geometric/algebraic in flavour, although the problem originates with a differential equation.

References include Philip Griffiths' article "Linearizing flows and a cohomological interpretation of Lax equations" American Journal of mathematics, 107 (1985), no 6, 1445–1484 (1986). and the section by Hitchin in the book "Integrable Systems – Twistors, Loop Groups, and Riemann Surfaces" by Hitchin, Segal and Ward. This area has wide applications in both differential geometry and mathematical physics.

iii) *Calibrations.* The notion of a calibrated geometry was introduced in a seminal paper by Harvey and Lawson. In these geometries, one studies special submanifolds that are globally area minimising (this is much stronger than the local condition that characterises minimal surfaces). The first non-classical example is special Lagrangian geometry, which plays an important part in mirror symmetry and is currently a hot research area. One can also use the octonions to define three more calibrated geometries, termed exceptional geometries due to their relationship with exceptional Lie groups. This area requires some background in differential geometry, such as that provided by MATH 3968.

iv) *Quaternionic Holomorphic Geometry.*

When studying the geometry of surfaces, one usually works locally as there are few global tools available. A couple of years ago it was observed that surfaces in S^4 could be studied more globally, using quaternionic analogs of standard complex analytic results. The quaternions enter the picture since S^4 is isomorphic to the quaternionic projective line, and one can use these tools to study surfaces in $\mathbb{R}^3$ simply by embedding $\mathbb{R}^3$ in S^4. This new theory is being used to study conformal immersions of surfaces, and in particular to attack the Willmore conjecture. A good reference is the book "Conformal geometry of surfaces in S^4 and quaternions" by Burstall, Ferus,Leschke, Pedit and Pinkall, available online at front.math.ucdavis.edu.

v) *Minimal Surfaces.* Physically, minimal surfaces model soap films: they locally solve the problem of finding the least area surface with a given boundary. They have been extensively studied and have a rich theory, with many interesting examples and generalisations. They are an active area of current research. There are various possibilities here for a project; ranging from the very explicit (some of the most exciting research here involves finding new examples), to the more theoretical. David Hoffman's expository article "The computer-aided discovery of new embedded minimal surfaces" in *The Mathematical Intelligencer* **9** no. 3 (1987), and Robert Osserman's book *A Survey of Minimal Surfaces* are good places in which to get a feel for this area.

Dr Alexander Fish – Carslaw 712

I am doing research in two fields – additive combinatorics and algebraic methods in wireless communication. I will be happy to supervise an honours thesis in any related topic. Some of the possible topics include:

i) Ergodic theory in Combinatorial Number Theory: Studying basics of Ergodic Theory, and subsequently, to write a thesis on Furstenberg's proof of Szemeredi theorem and Green-Tao proof of the theorem that primes contain arbitrary long arithmetic progressions.

ii) Additive Combinatorics and Freiman Theorem: This project will focus on one of the most important theorems of additive combinatorics — Freiman theorem. It roughly says that if a finite set of integers A has small doubling, i.e. the number of elements in A+A is bounded by a constant times the number of elements in A (constant is independent of A), then A has a structure similar to the structure of arithmetic progression. The thesis will include modern generalisations of Freiman theorem, namely, works of Breiilard-Green-Tao on approximate groups.

iii) Algebraic Constructions in Wireless Communication: One of the important objectives in modern digital signal processing (DSP) is a construction of sequences of large length having nice correlation properties. The best methods for providing such sequences are algebraic. The project will concentrate on construction of pseudo-random sequences based on exploring the symmetries of the vector space of sequences — the Heisenberg-Weil group of representations. The project will combine a purely mathematical study of modern algebraic methods in DSP with making computational experiments in MATLAB.

Dr James Parkinson – Carslaw 614

My main research interests lie at the confluent of algebra and geometry. My preference is for combinatorial techniques, attempting to understand complicated mathematical objects using simple combinatorial gadgets and tools. Please come and see me to discuss possible essay topics, with possible starting points including:

- *The incidence geometries of classical groups.* In this project you would discover how the (ubiquitous!) classical matrix groups over arbitrary fields act on beautiful 'point-line geometries' called *incidence geometries*.

- *Projective geometry: The Veblen-Young Theorem (and beyond).* The celebrated Veblen-Young Theorem classifies projective spaces when dimension is greater than 2. In your essay you could prove this beautiful and important theorem, and also investigate the zoo of non-classical examples in the critical dimension 2 case.

- *Random walks on Coxeter groups and hyperplane arrangements.* Representation Theory can be used to study probability theory on groups, with a famous example being the Bayer-Diaconis Theorem stating that 7 shuffles are required to adequately randomise a deck of cards. In this project you would discover the beauty of Coxeter groups and associated geometric configurations, and investigate how representation theory can be used to understand the long term behaviour of random processes on these objects.

- *Automata for Coxeter groups and surface groups.* The concept of an 'automatic structure' for a group has had a profound influence on computational group theory. This project would involve a detailed study of the beautiful class of 'Coxeter groups' and 'surface groups', with a focus on understanding and computing their automata. Applications include computing growth rates in these groups.

Professor Laurentiu Paunescu – Carslaw 721

I am interested in the applications of singularity theory to differential equations, and in using the combinatorics of Toric Modifications in investigating the equisingularity problem. My main research interests are:

i) *Singularities of complex and real analytic functions.*

ii) *Stratified Morse theory.*

iii) *Toric resolution of singularities.*

Professor Jacqui Ramagge – Carslaw 523

I work in both group theory and functional analysis. Within group theory, my main interest at the moment is in totally disconnected locally compact groups. These include automorphism groups of geometries and include all discrete groups as a subclass. They are a key ingredient in the classification of locally compact groups but very little was known about them until relatively recently. In functional analysis I have lately focussed my attention on C*-algebras and dynamical systems.

Specific examples of Honours projects could include:

i) Scale-multiplicative semigroups in totally disconnected, locally compact groups.

   This is one aspect of a large ongoing project on totally disconnected locally compact groups. This project will be an introduction to the rapidly growing general theory of totally disconnected locally compact groups, using specific new examples as the vehicle for the theory. Assumed knowledge would be linear algebra, group theory, and topology.

ii) C*-algebras and KMS states

   A C*-algebra is a closed algebra of continuous linear operators on complex Hilbert space. They are used as models in physics. The physical states and observables of a system are represented by unit vectors in the Hilbert space and self adjoint operators in the algebra respectively. The dynamical evolution of the system over time is embodied by a one-parameter group of transformations. KMS states (named after Kubo, Martin, and Schwinger) are equilibrium states of the system. Think of H_2O as having equilibrium states as steam, water, and ice. At any given point a system may have no equilibrium states, a unique equilibrium state, or a family of equilibrium states parametrised in some mathematically consistent way. There are phase transitions each time we change from one situation to another. This is a project in functional analysis. It assumes linear algebra and analysis (both real and complex).

I am also open to cosupervising projects in mathematical education that involve using mathematical and statistical techniques to analyse student performance.

iii) Applications of C*-algebras to quantum computing

I am interested in learning more about the mathematics underpinning quantum computing and it seems to me that the best way to do this is by finding someone to learn it with! This will involve looking at the relationship between C*-algebras and topological insulators. The project will assume linear algebra, analysis, and group theory to varying degrees depending on the particular direction we take.

Dr Anne Thomas – Carslaw 716

I would be happy to supervise an Honours essay on any topic related to my research interests, which include geometric group theory, rigidity and lattices in locally compact groups. Some sample topics are as follows, but these should be taken only as a guide.

i) *Hyperbolic groups.* One of the fundamental ideas in geometric group theory is to study finitely generated groups via the geometry of their associated Cayley graphs. A hyperbolic group is a group whose Cayley graph is “negatively curved” in an appropriate sense. Hyperbolic groups share many properties with fundamental groups of compact surfaces of genus ≥ 2. Possible directions include algorithmic properties, boundaries, connections with cube complexes, separability properties and the surface subgroup conjecture. Suggested reading: Bridson and Haefliger, *Metric Spaces of Non-Positive Curvature*.

ii) *Bass-Serre theory.* Group actions on trees may be encoded using the combinatorial data of a graph of groups. This beautiful theory has many applications to the study of group splittings, low-dimensional topology and lattices. Suggested reading: Serre, *Trees*.

iii) *Coxeter groups and nonpositive curvature.* Coxeter groups, which are generated by reflections, are ubiquitous in mathematics. Infinite Coxeter groups may be studied via their action on a contractible cell complex, the Davis complex, which satisfies an important combinatorial nonpositive curvature condition called the CAT(0) inequality. Suggested reading: Davis, *The Geometry and Topology of Coxeter Groups*.

iv) *Buildings.* Buildings are highly symmetric simplicial complexes that are associated to certain groups, and are used to study these groups. Various classes of buildings have been classified and there are important results relating their local to their global structure. Suggested reading: Ronan, *Lectures on Buildings*.

A/Prof Stephan Tillmann – Carslaw 710

I am available to supervise projects in geometry and topology, with an emphasis on low-dimensional objects, such as knots, surfaces, 3-dimensional and 4-dimensional spaces. I tend to use algebraic or combinatorial techniques, as well as synthetic geometric arguments. One of the aims of my research is to understand the structure of low-dimensional spaces and to obtain new invariants of these spaces.

Specific projects will be designed depending on your interests and experience. The aim of a thesis topics could be to understand a beautiful piece of theory; produce new examples; design or implement new algorithms to compute invariants; work out a discretisation of a smooth theory.

Dr Haotian Wu – 534

My research area is geometric analysis, which employs analytic tools such as partial differential equations to study problems in geometry and topology. I have worked on geometric flows such as Ricci flow and mean curvature flow, (moduli) space of Riemannian metrics, and geometry problems that arise in mathematical general relativity.

I look forward to supervising honours student(s). You honours essay topic could include:

i) **Existence of "nice" metrics**. A fundamental question in differential geometry is that if a given a manifold M carries a "nice" Riemannian metric. For example, an n-sphere quipped with the standard round sphere is nice in the sense that its curvatures are all the same positive constant. Two research topics in this direction are:

   - **Ricci flow**[1]: Here one deforms a metric g by $\partial_t g = -2\mathrm{Ric}(g)$, where $\mathrm{Ric}(g)$ is the Ricci curvature of g. Ricci flow is an efficient way to deform the metric into a "nice" one known as the Einstein metric. However, Ricci flow is nonlinear and tends to develop singularities in finite time, and singularity analysis poses a fundamental problem in the theory.
   - **Yamabe problem**: Here one seeks the existence of metric with constant scalar curvature in a given conformal class on a given manifold, e.g. the round metric on an n-sphere. This question can be formulated as an elliptic PDE, or can be approached by parabolic methods.

ii) **(Moduli) space of metrics**. Given a manifold M, denote by $\mathcal{M}$ the space of all Riemannian metrics on M. Then we can study the properties of the space $\mathcal{M}$. For example, is there a tangent space to $\mathcal{M}$ at a point (i.e. a metric)? If so, what is it? Or, is the space $\mathcal{M}$ path-connected? What about its higher homotopy groups? If two metrics g, g' are diffeomorphic, then (M, g) and (M, g') are geometrically indistinguishable. Thus, denote by $\mathfrak{D}$ the space of all diffeormphisms of M, we also study the moduli space $\mathcal{M}/\mathfrak{D}$.

iii) **Zoo of submanifolds**. Classical differential geometry studies curves and surfaces which are submanifolds of the flat Euclidean $\mathbb{R}^3$ (cf. MATH3968). One can generalise

[1] Ricci flow has been successful in proving Thurston's Geometrisation Conjecture of three manifolds and the Poincaré Conjecture, one of the "million-dollar maths problems".

to other ambient space (M, g) and study submanifolds of special properties such as *minimal* (zero mean curvature) and *constant mean curvature* (CMC) submanifolds. For example, Which CMC submanifolds exist in a given (M, g)? How many are there? Can we classify them? In this context, one can also study *mean curvature flow*.

iv) **Mathematical general relativity**. The space-time of our universe is studied using Riemannian geometry (space) and Lorentzian geometry (space-time) and contains many fascinating topics. To name a few: the mathematics of black holes is related to that of minimal surfaces in a space-time; the Positive Mass Theorem and its connection to scalar curvature; the Riemannian Penrose Inequality can be proved using the inverse mean curvature flow.

If you have other topics related to geometry, topology, or analysis in mind, please do not hesitate to email me so we could discuss the possibilities.

A/Prof Zhou Zhang – Carslaw 620

My primary research interest lies in differential geometry. Techniques from the theories of partial differential equation and several complex variables are frequently called for. The problems being considered often have strong algebraic geometry background. In the following, a few topics suitable for an honours essay are listed. If you already have your own topic(s) in mind, I'll be more than happy to discuss with you and provide suggestions if you wish.

i) *Futaki Invariant.* The goal is to look at some basic facts regarding this very important invariant in the study of Einstein metric. The understanding and justification of definition itself would already serve as a good way to introduce basics in complex differential geometry.

ii) *Ricci Flow and Maximum Principle.* Ricci flow and the complex version of it, Kähler-Ricci flow are topics of great interest in the recent years. Maximum Principle is, in principle, a very simple-minded tool in the study of differential equations. We could focus on Hamilton's Tensor Maximum Principle and provide some taste of how something so intuitive can go such a long way. In this topic, we would start with some introduction to Riemannian geometry if needed.

iii) *Introduction to Algebraic Curves.* Here I am using the title of the book by Griffiths. It provides a few very good topics to work on for beginners in algebraic geometry, for example, the Riemann-Roch Theorem in Chapter III. The discussion of these low dimensional objects naturally serves as a motivation in the study for general dimension.

iv) *De Rham Theorem and Hodge Theorem.* This topic serves as an introduction to differential geometry for manifolds. For the most part, we could follow the nice textbook "Foundations of differentiable manifolds and Lie groups" by Warner, and add some more in the Kähler setting, if time allows, where things come together in a very nice way.

v) *Characteristic Classes.* Here I am using the title of the famous book by Milnor. From one point of view, it provides a topological (and intuitive) way to describe objects of great interests in differential geometry.

5.4 Non-Linear Analysis Research Group

A/Prof Florica Cîrstea – Carslaw 719

My main research interests concern nonlinear partial differential equations (PDEs). In this area there are many important topics that can be treated using various modern approaches. I would be happy to supervise such topics from the theory of both linear and nonlinear PDEs. Some specific projects are provided below, but students are encouraged to negotiate the topic for a best match.

i) The theory of linear PDEs relies on functional analysis and relatively easy energy estimates to prove the existence of weak solutions to various linear equations. The proper setting for the study of many linear and nonlinear PDEs via energy methods is provided by the so-called Sobolev spaces. If we require the solutions of a given PDE to be very regular, we would usually have a very hard time to find them. A more profitable approach is to consider the issue of existence of solutions separately from the smoothness (or regularity) problems. The idea is to introduce a new concept of solution (weak solution) that does not have too much smoothness so that we could establish its existence, uniqueness and continuous dependence of the given data. Various PDEs could be treated in this way and this is, possibly, the best we can do in many cases. For others, we could hope to prove that our weak solution becomes smooth enough to be deemed as a classical solution. This leads to the issue of regularity of weak solutions, which relies on many intricate estimates.

   Possible topics. Investigating the solvability of uniformly elliptic, second-order PDEs, subject to prescribed boundary conditions using two essentially different techniques: energy methods within Sobolev spaces and maximum principle methods. The energy methods can be expanded to a variety of linear PDEs characterizing evolutions in time. This broadens the class of PDEs to include the heat equation and more general second-order parabolic PDEs, as well as the wave equation (and general second-order hyperbolic PDE).

ii) The theory of nonlinear PDEs is far less unified in its approach compared with the linear one. Variational methods provide one of the most useful and accessible of the approaches for nolinear PDEs. Other techniques are also available for nonlinear elliptic and parabolic PDEs such as the monotonicity and fixed-points methods, as well as other devices involving the maximum principle. The study of such techniques would make interesting essay topics.

A/Prof Daniel Daners – Carslaw 715

Areas of interest:

i) *Partial differential equations (linear or nonlinear).*

ii) *Ordinary differential equations (linear or nonlinear).*

iii) *Bifurcation theory.*

iv) *Analytic semigroup theory and abstract evolution equations.* (This is a theory of "ordinary differential equations" in infinite dimensional spaces with applications to partial differential equations.)

Please see me to negotiate a topic of your interest or for suggestions for specific projects related to the above areas.

Dr Leo Tzou – Carslaw 615

Probing the Earth and the Universe with Microlocal Analysis

This essay is under the supervisor's ARC Discovery Project of the same name and has the potential to continue to a PhD position partially funded by ARC. The student will have opportunities to attend, subject to availability, the workshop organized by the supervisor at the Banff International Research Station and Fudan Conference on Microlocal Analysis. During the first semester student will also have opportunity to interact with prominent mathematicians in the field who are visiting through the School's International Visitor's Program.

This project aims to use the theory of microlocal analysis to determine the amount of information one can recover about the earth and the universe by making observations on wave propagation. In addition to applications to seismic imaging and cosmology, this project will generate new knowledge in the field of differential geometry and dynamical systems. This will be accomplished by formulating the tomography problem in the language of differential geometry and introduce new analysis techniques to study them. Expected outcome of this project will be new rigidity type results in Lorentzian and Riemannian geometry. There is also the potential for downstream impacts in seismic and cosmological imaging.

Image Identification with Incomplete Data

This essay is under the supervisor's ARC Discovery Project *"Inverse Problems with Partial Data"* and has the potential to continue to a PhD position partially funded by ARC. The student will have opportunities to attend, subject to availability, the workshop organized by the supervisor at the Banff International Research Station and Fudan Conference on Microlocal Analysis. During the first semester student will also have opportunity to interact with prominent mathematicians in the field who are visiting through the School's International Visitor's Program.

This project aims to use mathematics, in particular the theory of micro-local analysis, to determine the amount of measurements one needs in order to reconstruct an image by some of the tomography methods commonly used in medical imaging. Expected outcomes of this project include showing that an arbitrarily small set of boundary measurements is sufficient to reconstruct the coefficients of various important partial differential equations such as

Schrodinger equation, Dirac operators, and Maxwell equations. In addition to providing a theoretical foundation upon which one can build numerical algorithms, this project will also provide the missing link between inverse problems and unique continuation theory. The downstream impact of this research will lead to more efficient and accurate tomography methods which can be implemented in a range of imaging applications.

Pattern Formation Through Geometric Microlocal Analysis

This essay is under the supervisor's ARC Discovery Project *Probing the Earth and the Universe with Microlocal Analysis* and has the potential to continue to a PhD position partially funded by ARC and jointly supervised by applied mathematicians at Macquarie University. There can also be a computational component to the project.

The student will have opportunities to attend, subject to availability, the workshop organized by the supervisor at the Banff International Research Station and Fudan Conference on Microlocal Analysis. During the first semester student will also have opportunity to interact with prominent mathematicians in related fields who are visiting through the School's International Visitor's Program.

We model the effect of drought on vegetation patterns on variable terrains. Mathematically these are described by the dynamics and stability of localized patterns in reaction-diffusion systems on surfaces of non-constant curvature. In systems such as the (rescaled) Schnakenberg model which often serves as a prototype for such studies, the behaviour of localized patterns depend on the asymptotic of the singularity for the Green's function of the Helmholtz operator near the singularity. In the general geometric setting, it is often difficult to derive an explicit formula for the Green's function (or even to compute it numerically), which is why existing theory can only treat simple geometries such as the disk and sphere.

In the language of microlocal analysis, however, this difficulty becomes a standard parametrix construction. This recent breakthrough is allowing us to study pattern formation on terrains which were previously inaccessible.

Extraordinaire comme les mathématiques vous aident à
vous connaître.

Samuel Beckett, *Molloy*

Chapter 6

The Talk

6.1 General remarks

Before the essay is submitted at the end of Second Semester, each student gives a talk on their essay project. The talks will usually take place in the week before the mid-semester break.

The aim of the talk is to provide training in the explanation to others of the purpose and nature of a project, within definite time limits; twenty minutes for each talk, plus five minutes for questions.

All members of the Department, Fourth Year and postgraduate students are invited to the Fourth Year talks.

The talk is worth 5% of your essay mark.

6.2 Preparing the talk

The purpose of your talk is to convey to your *fellow students* (and the academic staff) what you are working on. They probably know very little about your essay topic; this comment may also apply to the academic staff. Do not make the talk too long or ambitious. Aim to convey the essence of your project to the audience rather than trying to impress the audience; after all, it is unlikely that you can cover the whole of your project in 20 minutes!.

The key to giving a successful mathematical talk is: "Keep it simple!" *One* idea, illustrated by one or two examples, is enough for a good talk. A special case often conveys more than a general, all-encompassing theorem. For example, to give the flavour of general fields, a detailed study of a simple, but unfamiliar field, such as GF(9), might be appropriate.

Keep in mind that the audience is swept along with you and that they cannot go back to earlier stages of your talk like when they are reading an article. You are not giving a lecture, so although some definitions may be appropriate, lengthy technical proofs should be avoided. It is also not a good idea to over-develop the theory at the expense of examples: a well-chosen example is worth ten thousand theorems. Finally, try and relate your content to other areas of mathematics or applications; this can make the talk much more interesting for the general audience.

You should aim your talk at a general mathematical audience and *avoid* directing it at the odd specialist in your topic in the audience. Thus a good talk is judged by one criterion: you have given the audience, especially your fellow fourth year students, a good idea of your project and its significance.

Discuss the talk with your supervisor.

Having chosen the topic for your talk, prepare a written outline. Some people write their talk out in full, while others prefer to use only a written outline and allow improvisations. As it is probably your first talk of this kind, it is advisable to do a full dress rehearsal the previous evening; so find a blackboard or a projector and go through the complete talk. This will help you in judging the timing of your talk properly: it takes much longer to say things than you probably realize. If you can, find a sympathetic listener to give you feedback. Your listener does not have to be mathematically literate: a good talk is almost as much about theatre and presentation as it is about mathematics.

6.3 Slide Talks

Decide if the use of Beamer, PowerPoint or an overhead projector is appropriate. This allows preparation of complicated figures or tables ahead of time, or the inclusion of photocopies of published material in your exposition. Beware, however, that although the speaker can by this means pass a vast amount of information before the eyes of the audience very quickly, the audience will probably not take it all in. It is important either to write clearly and in large letters and to refer explicitly to each line (say by gradually revealing line-by-line) or, in the case of a diagram or complicated formula, to allow your audience time to absorb its detail.

If you are going to use LaTeX to create slides then the use of the Beamer package is recommended.

Life is good for only two things, discovering mathematics
and teaching mathematics.

Siméon Poisson

Chapter 7

Your Future and Mathematics

As a fourth year student you are a member of the mathematics department and you should take advantage of the facilities it offers. The University of Sydney has one of the top mathematics research departments in the country, and it ranks very highly internationally in several areas. There are also a number of prominent international (short and long term) visitors to the department who give seminar talks within the department. It pays to keep an eye on `scnews` (the School's web based bulletin board), for upcoming seminar announcements.

The academic staff, the many postdocs and the visitors to the department are all usually very happy to talk mathematics talk with interested students: all you have to do is find the courage to ask!

Fourth year students are also very welcome to join the staff and postgraduates in the use of the tea room; this can be a good place to meet other people in the department.

7.1 The colloquium and other seminars

Many Fridays during the year, a Colloquium is held at 2:30 pm at the University of Sydney or at 2:00 p.m. in the 'Red Centre' UNSW. Topics vary, but the intention is to provide a one-hour talk on a subject of contemporary mathematical interest to a general audience. Fourth Year students are encouraged to attend the Colloquium and indeed are welcome to any seminar run in the Department. For a schedule of upcoming seminars, see scnews and the seminar websites that are linked to from the main school webpage.

7.2 After fourth year, what then?

Recent graduates have found employment in a wide variety of occupations: computer related jobs, teaching (University or School), positions in insurance and finance. To find out more about where maths can take you:

http://www.careers.usyd.edu.au

http://www.amsi.org.au
http://www.austms.org.au/Jobs

Here we shall just outline briefly the postgraduate degree options. For more information consult the departments web pages.

7.3 Higher degrees

A result of II-2 or better is the minimum requirement for entry into a higher degree at Sydney. However it should be noted that one should not normally contemplate continuing without a result of at least II-1. Anyone intending to undertake a higher degree should consult with the Mathematics Postgraduate Coordinator (Prof Rutkowski Marek), as soon as possible. The usual practice is to enrol for an M.Sc. in the first instance and later to convert to a Ph.D. if it is desired to continue.

Information on scholarships for postgraduate study can be found at
http://www.maths.usyd.edu.au/maths/u/PG/.

7.4 Scholarships and other support

Scholarships, prizes and travel grants are available both for study at Sydney and for study elsewhere. Full details can be found in the University Calendar and from the Scholarships Office (Administration Building). Intending applicants should obtain application forms from the Scholarships Office as soon as possible. *The closing dates for some scholarships can be as early as September.*

If you are considering further study at an Australian University, you should apply for an Australian Postgraduate Research Award (even for an M.Sc. by coursework). For study at a university in Britain or Canada, apply for a University of Sydney travelling Scholarship and also apply to the chosen university for employment as a Graduate Assistant.

7.5 Further study in another subject

As mentioned in the introduction to this booklet, it is quite possible to do Fourth Year Pure Mathematics and then continue with a higher degree in another subject. Within Australia, prerequisites vary from university to university and department to department, and for those intending to follow this path it is advisable to consult with the department concerned to determine an appropriate choice of fourth year topics. If you are intending to continue with postgraduate studies in another field *outside* Australia, do check prerequisites. Provided you have done third year courses in the subject at Sydney, you will *probably* not encounter significant problems over prerequisites.

It has long been an axiom of mine that the little things are infinitely the most important.

Sherlock Holmes, *A Case of Identity*

Appendix A

Instructions on Preparing the Manuscript

Essays must be typed using LaTeX (or TeX), or a commercial word processing program such as word. Amongst professional mathematicians LaTeX has become the standard; it produces better quality output than any word processing programs program—at least when it comes to mathematics. The downside to LaTeX is that it takes some time to learn.

The fourth year coordinator will give an introduction to using TeX and LaTeX before the beginning of second semester. For those wishing to use LaTeX Prof Mathas has written a LaTeX class file that takes care of the basic layout of the essay; for information, as well as some basic tips on how to use LaTeX, see http://www.maths.usyd.edu.au/u/mathas/courses/pm4/.

See http://www.maths.usyd.edu.au/u/SMS/texintheschool.html for links to TeX and LaTeX documentation available on the School's website. The LaTeX package amsmath and BiBTeX are perhaps the most important. (Note that Prof Mathas' class file preloads the amsmath package, which is essential for adequately typesetting mathematics in a LaTeX document. BiBTeX is used for automatically including a bibliography in a LaTeX document.)

If you decide not to use this LaTeX class file, then your document must nevertheless satisfy the following requirements.

1. A margin of at least 2.5cm must be left at the top, bottom, left- and right-hand side of each page. The margin is determined by the last letter or character in the longest line on the page.
2. All pages must be numbered (in a consistent way), except for the title page.
3. Avoid excessive use of footnotes. They are rarely necessary in mathematics.
4. Diagrams should be created using appropriate software; check with your supervisor first if you intend to use hand drawn diagrams.
5. Theorems, Propositions, and such like, should be labelled consistently throughout the document.
6. The font size must be 12pt.