1. Complete the following definitions.
   (a) A connected graph is Eulerian if it has a walk which returns to its starting vertex and uses every edge exactly once.
   (b) A connected graph with \( \geq 3 \) vertices is Hamiltonian if it has a walk which returns to its starting vertex and uses every vertex exactly once.

2. Consider the following graph \( G \).

   \[
   \begin{array}{c}
   a \quad b \\
   \quad e \quad f \quad h \\
   d \quad c
   \end{array}
   \]

   (a) Its degree sequence is \( (2, 3, 3, 3, 4, 4) \)
   (b) Is it regular? \( \boxed{\text{no}} \)
   (c) Is it Eulerian? \( \boxed{\text{no}} \)
   (d) Is it Hamiltonian? \( \boxed{\text{yes}} \)
3. Consider the graphs \( P_2, P_3, P_4, C_3, C_4, K_4, K_5 \).

(a) Which of them are regular?

(b) Which of them are Eulerian?

(c) Which of them have an Eulerian trail?

(d) Which of them are Hamiltonian?

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Regular</th>
<th>Eulerian</th>
<th>Eulerian Trail</th>
<th>Hamiltonian</th>
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</thead>
<tbody>
<tr>
<td>( P_2, C_3, C_4, K_4, K_5 )</td>
<td>( P_2, C_3, C_4, K_4, K_5 )</td>
<td>( C_3, C_4, K_5 )</td>
<td>( P_2, P_3, P_4 )</td>
<td>( C_3, C_4, K_4, K_5 )</td>
</tr>
</tbody>
</table>

4. For each of the following sequences, say whether it is graphic or not graphic.

(a) \( (0, 0, 0, 0) \) graphic

(b) \( (0, 0, 1, 1) \) graphic

(c) \( (0, 1, 1, 1) \) not graphic

(d) \( (0, 0, 2, 2) \) not graphic

(e) \( (3, 3, 3, 3) \) graphic

(f) \( (2, 2, 2, 2, 2, 2, 2) \) graphic

(g) \( (3, 3, 3, 3, 3, 3, 3, 3) \) graphic

(h) \( (0, 0, 1, 1, 2, 2, 3, 3, 3, 3) \) graphic

(i) \( (1, 3, 3, 3, 5, 5) \) not graphic
5. For each of the following graphs, list the vertices of an Eulerian circuit or Eulerian trail. (In all cases there is more than one correct answer.)

(a) $a, b, d, c, a, d$

(b) $b, a, d, c, b, d, e, a, c, e$

(c) $a, d, e, a, c, d, b, e, f, a$
6. For each of the following statements, write T for true or F for false.

(a) If a graph has degree sequence (1, 2, 2, 2, 3),
the degree-3 vertex must be adjacent to all the degree-2 vertices. \[ \text{F} \]

(b) Any regular connected graph with \( \geq 3 \) vertices is Hamiltonian. \[ \text{F} \]

(c) \( K_{2,2} \) is isomorphic to \( C_4 \). \[ \text{T} \]

(d) A graph is Hamiltonian if and only if it contains a cycle. \[ \text{F} \]

(e) If \( v \) is a vertex of a Hamiltonian graph \( G \), then \( G - v \) is connected. \[ \text{T} \]

(f) If \( G - v \) is connected for every vertex \( v \), then \( G \) is Hamiltonian. \[ \text{F} \]

(g) If \( v \) and \( w \) are non-adjacent vertices of a Hamiltonian graph \( G \)
with \( n \) vertices, then \( \deg(v) + \deg(w) \geq n \). \[ \text{F} \]

(h) If, for every pair of vertices \( v \) and \( w \) of a connected graph \( G \)
with \( n \geq 3 \) vertices, \( \deg(v) + \deg(w) \geq n \), then \( G \) is Hamiltonian. \[ \text{T} \]