1. In each part, determine whether the two pictures represent the same isomorphism class of graphs. If the answer is yes, label the vertices so that they become two pictures of the same graph. If the answer is no, give a reason why.

(a)

(b)

(c)

(d)

2. Draw the complements of the following two graphs. Are these complements isomorphic to each other?
3. Translate each of the following problems into a problem about a graph. You don’t need to solve the problems (in fact you can’t, because the specific information required is lacking): just say how to define the graph, and what the problem is asking in terms of that graph.
   (a) The communications between the computers in a network go through various ethernet cables, each of which joins two computers. How many ethernet cables have to be removed before the computers are no longer all able to communicate with each other?
   (b) An airline flies various routes between cities around the world. Is it possible to construct a round-trip itinerary which visits every city that the airline flies to, without ever repeating a route?
   (c) A domino is a rectangular tile displaying \( i \) dots near one end and \( j \) dots near the other, where \( i, j \in \{0, 1, \ldots, 6\} \). Suppose we have some dominoes which don’t include any ‘doubles’ (that is, \( i \) is always different from \( j \)) and which are all different (so no set \( \{i, j\} \) appears more than once). Is it possible to lay these dominoes out in a long line so that whenever two ends touch, they have the same number of dots?
   *(d) Of the various animals in a zoo, some pairs are compatible with each other and some pairs aren’t. What is the smallest number of cages required to house all the animals, if the only constraint is that any two animals in the same cage are compatible with each other?*

4. Give an example of a graph which is isomorphic to its own complement.

5. Fix an integer \( n \geq 2 \). Let \( i \neq j \) be two vertices of the complete graph \( K_n \).
   (a) How many walks of length 2 are there in \( K_n \) from \( i \) to \( j \)?
   (b) How many walks of length 3 are there in \( K_n \) from \( i \) to \( j \)?
   *(c) Let \( a_\ell \) be the number of walks of length \( \ell \) in \( K_n \) from \( i \) to \( j \). Find a closed formula for \( a_\ell \) in terms of \( n \). (*Hint: show that the sequence \( a_0, a_1, a_2, \ldots \) satisfies a recurrence relation.)*

6. In this question, \( G \) denotes a connected graph with at least 3 vertices. A vertex \( v \) of \( G \) is called a cut vertex if the graph \( G - v \) obtained by removing it is not connected.
   (a) If \( G \) is the cycle graph \( C_n \), how many cut vertices does it have?
   (b) If \( G \) is the path graph \( P_n \), how many cut vertices does it have?
   (c) Prove that if \( G \) contains a bridge, then it contains a cut vertex. Is the converse true?
   *(d) Prove that \( v \) is a cut vertex of \( G \) if and only if there are two other vertices \( u \) and \( w \) of \( G \) such that \( v \) belongs to every path between \( u \) and \( w \).*

7. Prove that, for any graph \( G \) with at least one vertex, either \( G \) is connected or its complement \( \overline{G} \) is connected.