1. Consider the recurrence relation \( a_n = 4a_{n-1} - 4a_{n-2} + 3n + 2 \), for \( n \geq 2 \).
   (a) If \( p_n \) is a particular solution of this recurrence, the general solution is

   \[
   a_n = \]

   where \( b_n \) is a general solution of the homogeneous recurrence relation, i.e. 
   \( b_n = 4b_{n-1} - 4b_{n-2} \).
   (b) The characteristic polynomial of the homogeneous recurrence is \( x^2 - 4x + 4 \).
   Hence

   \[
   b_n = \]

   (c) A particular solution of the form \( p_n = An + B \) is

   \[
   p_n = \]

   (d) Find the solution of the original recurrence when \( a_0 = 15 \), \( a_1 = 21 \):

   \[
   a_n = \]

2. Write down the general solution of each of the following recurrence relations, by finding a particular solution of the stated form.
   (a) \( a_n = 3a_{n-1} + 2 \) for \( n \geq 1 \); \( p_n = A \).

   \[
   a_n = \]

   (b) \( a_n = 2a_{n-1} + n + 1 \) for \( n \geq 1 \); \( p_n = An + B \).

   \[
   a_n = \]

   (c) \( a_n = 3a_{n-1} - 2^n \) for \( n \geq 1 \); \( p_n = A2^n \).

   \[
   a_n = \]
3. If \( A(z) = \sum_{n=0}^{\infty} a_n z^n \) and \( B(z) = \sum_{n=0}^{\infty} b_n z^n \) are two formal power series, complete the definitions of the following formal power series:

\[
A(z) + B(z) =
\]

\[
A(z)B(z) =
\]

\[
A'(z) =
\]

4. For each of the sequences (a)–(g), write the number (i)–(vii) of its generating function.

(a) 1, 2, 2^2, 2^3, \cdots

(b) 1, 1, 1, 1, \cdots

(c) 3, 2, 1, 0, 0, 0, \cdots

(d) 1, 2, 3, 4, 5, \cdots

(e) 0, 1, 2, 3, 4, \cdots

(f) 0, 0, 1, 2, 3, 4, \cdots

(g) 2, 3, 4, 5, 6, \cdots

(i) \( \frac{1}{1 - z} \)

(ii) \( \frac{1}{(1 - z)^2} \)

(iii) \( \frac{1}{1 - 2z} \)

(iv) \( \frac{z^2}{(1 - z)^2} \)

(v) \( \frac{3 + 2z + z^2}{1 - z} \)

(vi) \( \frac{z}{(1 - z)^2} \)

(vii) \( \frac{2 - z}{(1 - z)^2} \)
5. For each of the following power series, give a formula for the coefficient of \( z^n \).

(a) \( \frac{3}{1 - z} \)

(b) \( \frac{1}{1 - 3z} \)

(c) \( \frac{3}{1 + z} \)

(d) \( \frac{3}{(1 + z)^2} \)

(e) \( \frac{1}{(1 - 3z)^2} \)

6. Give formulas for the following power series in terms of \( A(z) = \sum_{n=0}^{\infty} a_n z^n \).

(a) \( \sum_{n=0}^{\infty} (a_0a_n + a_1a_{n-1} + \cdots + a_{n-1}a_1 + a_n a_0) z^n \)

(b) \( \sum_{n=0}^{\infty} (a_0 + a_1 + \cdots + a_n) z^n \)

(c) \( \sum_{n=0}^{\infty} (n + 1) a_{n+1} z^n \)

(d) \( \sum_{n=0}^{\infty} \left( \frac{a_0}{n!} + \frac{a_1}{(n-1)!} + \cdots + \frac{a_{n-1}}{1!} + \frac{a_n}{0!} \right) z^n \)
7. For each of the following statements, write T for true or F for false.

(a) The sequence $3, -3, 3, -3, \cdots$ has generating function $\frac{3}{1 + z}$.  
(b) $3 + z$ is an example of a formal power series.
(c) $\sum_{n=1}^{\infty} a_{n-1} z^n = \sum_{n=0}^{\infty} a_n z^{n+1}$. 
(d) $\frac{1}{1 + 3z} = \sum_{n=0}^{\infty} 3^n z^n$.
(e) $\frac{1}{(1 - 2z)^2} = \sum_{n=0}^{\infty} (n + 1)2^n z^n$.
(f) $\frac{1}{(1 - z)^3} = \sum_{n=0}^{\infty} \binom{n}{2} z^n$.
(g) If $\sum_{n=0}^{\infty} a_n z^n = \frac{z}{1 - 3z}$, then $a_{900} = 3^{901}$.
(h) If $A'(z) = 0$, then $A(z) = a_0$ for some constant $a_0$.
(i) For $n \geq 1$, the coefficient of $z^n$ in $\frac{4 - 6z}{1 - 2z}$ is $2^n$.
(j) For $n \geq 2$, the coefficient of $z^n$ in $\frac{1 - z^2}{1 - z}$ is 1.