1. Consider the recurrence relation \( a_n = 4a_{n-1} - 4a_{n-2} + 3n + 2 \), for \( n \geq 2 \).
   
   (a) If \( p_n \) is a particular solution of this recurrence, the general solution is
   \[
   a_n = b_n + p_n
   \]
   where \( b_n \) is a general solution of the homogeneous recurrence relation, i.e.
   \[ b_n = 4b_{n-1} - 4b_{n-2} \].
   
   (b) The characteristic polynomial of the homogeneous recurrence is \( x^2 - 4x + 4 \).
   Hence
   \[
   b_n = C_1 2^n + C_2 n 2^n \]
   for some constants \( C_1, C_2 \).
   
   (c) A particular solution of the form \( p_n = An + B \) is
   \[
   p_n = 3n + 14
   \]
   
   (d) Find the solution of the original recurrence when \( a_0 = 15 \), \( a_1 = 21 \):
   \[
   a_n = 2^n + n 2^n + 3n + 14
   \]

2. Write down the general solution of each of the following recurrence relations, by finding a particular solution of the stated form.
   
   (a) \( a_n = 3a_{n-1} + 2 \) for \( n \geq 1 \); \( p_n = A \).
   \[
   a_n = C3^n - 1 \text{ for some constant } C
   \]
   
   (b) \( a_n = 2a_{n-1} + n + 1 \) for \( n \geq 1 \); \( p_n = An + B \).
   \[
   a_n = C2^n - n - 3 \text{ for some constant } C
   \]
   
   (c) \( a_n = 3a_{n-1} - 2^n \) for \( n \geq 1 \); \( p_n = A2^n \).
   \[
   a_n = C3^n + 2^{n+1} \text{ for some constant } C
   \]
3. If \( A(z) = \sum_{n=0}^{\infty} a_n z^n \) and \( B(z) = \sum_{n=0}^{\infty} b_n z^n \) are two formal power series, complete the definitions of the following formal power series:

\[
A(z) + B(z) = \sum_{n=0}^{\infty} (a_n + b_n) z^n
\]

\[
A(z)B(z) = \sum_{n=0}^{\infty} \left( \sum_{m=0}^{n} a_m b_{n-m} \right) z^n
\]

\[
A'(z) = \sum_{n=0}^{\infty} (n+1)a_{n+1} z^n
\]

4. For each of the sequences (a)–(g), write the number (i)–(vii) of its generating function.

(a) 1, 2, 2^2, 2^3, · · · (iii) (f) 0, 0, 1, 2, 3, 4, · · · (iv)

(b) 1, 1, 1, 1, · · · (i) (g) 2, 3, 4, 5, 6, · · · (vii)

(c) 3, 2, 1, 0, 0, 0, · · · (v) (i) \( \frac{1}{1-z} \) (iv) \( \frac{z^2}{(1-z)^2} \)

(ii) \( \frac{1}{(1-z)^2} \) (v) \( 3+2z+z^2 \)

(iii) \( \frac{1}{1-2z} \) (vi) \( \frac{z}{(1-z)^2} \)

(d) 1, 2, 3, 4, 5, · · · (ii) (vii) \( \frac{2-z}{(1-z)^2} \)

(e) 0, 1, 2, 3, 4, · · · (vi)
5. For each of the following power series, give a formula for the coefficient of $z^n$.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\frac{3}{1-z}$</td>
</tr>
<tr>
<td>(b)</td>
<td>$\frac{1}{1-3z}$</td>
</tr>
<tr>
<td>(c)</td>
<td>$\frac{3}{1+z}$</td>
</tr>
<tr>
<td>(d)</td>
<td>$\frac{3}{(1+z)^2}$</td>
</tr>
<tr>
<td>(e)</td>
<td>$\frac{1}{(1-3z)^2}$</td>
</tr>
</tbody>
</table>

6. Give formulas for the following power series in terms of $A(z) = \sum_{n=0}^{\infty} a_n z^n$.

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\sum_{n=0}^{\infty} (a_0a_n + a_1a_{n-1} + \cdots + a_{n-1}a_1 + a_na_0)z^n$</td>
</tr>
<tr>
<td>(b)</td>
<td>$\sum_{n=0}^{\infty} (a_0 + a_1 + \cdots + a_n)z^n$</td>
</tr>
<tr>
<td>(c)</td>
<td>$\sum_{n=0}^{\infty} (n+1)a_{n+1}z^n$</td>
</tr>
<tr>
<td>(d)</td>
<td>$\sum_{n=0}^{\infty} \left(\frac{a_0}{n!} + \frac{a_1}{(n-1)!} + \cdots + \frac{a_{n-1}}{1!} + \frac{a_n}{0!}\right)z^n$</td>
</tr>
</tbody>
</table>
7. For each of the following statements, write T for true or F for false.

(a) The sequence 3, −3, 3, −3, · · · has generating function \( \frac{3}{1 + z} \). \hspace{1cm} T

(b) 3 + z is an example of a formal power series. \hspace{1cm} T

(c) \( \sum_{n=1}^{\infty} a_{n-1} z^n = \sum_{n=0}^{\infty} a_n z^{n+1} \). \hspace{1cm} T

(d) \( \frac{1}{1 + 3z} = \sum_{n=0}^{\infty} 3^n z^n \). \hspace{1cm} F

(e) \( \frac{1}{(1 - 2z)^2} = \sum_{n=0}^{\infty} (n + 1)2^n z^n \). \hspace{1cm} T

(f) \( \frac{1}{(1 - z)^3} = \sum_{n=0}^{\infty} \binom{n}{2} z^n \). \hspace{1cm} F

(g) If \( \sum_{n=0}^{\infty} a_n z^n = \frac{z}{1 - 3z} \), then \( a_{900} = 3^{901} \). \hspace{1cm} F

(h) If \( A'(z) = 0 \), then \( A(z) = a_0 \) for some constant \( a_0 \). \hspace{1cm} T

(i) For \( n \geq 1 \), the coefficient of \( z^n \) in \( \frac{4 - 6z}{1 - 2z} \) is \( 2^n \). \hspace{1cm} T

(j) For \( n \geq 2 \), the coefficient of \( z^n \) in \( \frac{1 - z^2}{1 - z} \) is 1. \hspace{1cm} F