This quiz lasts 30 minutes. There are a total of 15 marks.

Answers must be written in pen in the boxes provided. Anything written in pencil or written outside the boxes will not be marked.

Your answers can be given in the form of an expression such as \(4 \times 3 \times \left(\frac{1}{2}\right)\) or \(S(9,5)\), if you wish, as long as it fits in the answer box – i.e. you are not required to find the numerical value.
1. Complete the following definitions and formulations of theorems.

(a) A permutation of a set \( X \) is

\[
\text{a bijection } f : X \rightarrow X
\]

(b) The number of solutions of the equation \( y_1 + y_2 + \cdots + y_p = m \)
where all \( y_i \) are natural numbers is

\[
\binom{m + p - 1}{m}
\]

(c) The binomial coefficients satisfy the recurrence relation

\[
\binom{m}{r} = \binom{m-1}{r-1} + \binom{m-1}{r}
\]

(3 marks)

2. Consider the sequences of 18 zeros and 6 ones. What is the number of

(a) all such sequences?

\[
\binom{24}{6}
\]

(b) such sequences where the 6 ones are not all together?

\[
\binom{24}{6} - 19
\]

(c) such sequences where no two ones are next to each other?

\[
\binom{19}{6}
\]

(3 marks)
3. How many ways are there to put 18 different balls in 7 boxes $A_1, \ldots, A_7$ so that

(a) no box is left empty

(b) only three

of the seven boxes are left empty

(c) each nonempty box

contains only six balls

\[ 7! \cdot S(18, 7) \]

\[ \binom{7}{3} \cdot 4! \cdot S(18, 4) \]

\[ \binom{7}{3} \cdot \left( \begin{array}{c} 18 \\ 6, 6, 6 \end{array} \right) \]

(3 marks)

4. How many ways are there to give 13 identical candies to 4 boys and 4 girls so that

(a) no boy misses out?

\[ \binom{16}{7} \]

(b) no one misses out and no girl gets

more candies than any boy?

60

(2 marks)
5. Consider the recurrence relation $a_n = -11 a_{n-1} - 30 a_{n-2}$, for $n \geq 2$.

(a) What are the roots of its characteristic polynomial? \[ x = -5, -6 \]

(b) Write down the general solution:

\[ a_n = C_1 (-5)^n + C_2 (-6)^n \text{ for some constants } C_1, C_2 \]

(c) Give the solution when $a_0 = 2$ and $a_1 = -11$:

\[ a_n = (-5)^n + (-6)^n \]

(3 marks)

6. Consider the recurrence relation $a_n = r_1 a_{n-1} + r_2 a_{n-2} + r_3 a_{n-3}$, for $n \geq 3$. The root of its characteristic polynomial is $x = 5$ of multiplicity 3. Write down the general solution:

\[ a_n = C_1 5^n + C_2 n 5^n + C_3 n^2 5^n \text{ for some constants } C_1, C_2, C_3 \]

(1 mark)

END OF QUIZ.