THE UNIVERSITY OF SYDNEY
MATH2070 AND MATH2970

Optimisation and Financial Mathematics 2009

Lecturers : Bob Crossman and Georg Gottwald

Due 5pm Wednesday September 16th

MATH2070: Do questions 1, 2, 3 and **4**
MATH2970: Do questions 1, 2, 3, and **5**

Submit your assignment in the designated labelled box low down in the cabinet next to Room 623 in the Carslaw Building. Please use a wallet style folder and do not submit the assignment before Wednesday September 9th. The computer printout of your computing solutions must contain the banner indicating your user name. Any blank lines and errors should be edited out.

Formulate all Questions as Linear Programming problems:

Set up the models and return your answers using words and fully explain your reasoning in all questions including those (Q3, Q4) where you must also include the accompanying computer printout. Please write legibly in ink and make life as pleasant as possible for the marker.

Solve Q1 graphically. Use the Simplex method for the other questions
Solve Question 2 by hand. Use Matlab in Q3 and Q4 (MATH2070 only). Solve Q5 (MATH2970 only) by hand.

1. A small vineyard consists of two fields on either side of a creek, each planted with three varieties of grapes. It costs $300 per day to harvest the left field, obtaining in that time 1 tonne of cabernet grapes, 1.5 tonnes of shiraz grapes and 0.75 tonnes of pinot grapes. The right field is on a hill and more expensive to harvest, at a cost of $500 per day, but it yields in that time 5 tonnes of cabernet, 1.5 tonnes of shiraz and 0.25 tonnes of pinot grapes. The owner receives only one order, for 10 tonnes of cabernet, 7.5 tonnes of shiraz and 2.25 tonnes of pinot each season.

(a) How many days per season should each field be worked so as to fill this order as cheaply as possible?
(b) How many harvested tonnes of each grape variety will be surplus to requirements?
Solve Question 1 graphically.

2. A tea trader sells ``*Tip Top*" and ``*No Name*" brands of tea. Both are blended from three grades of tea (*A*, *B* and *C*) which she imports. The composition of the blends comprising each brand is:

$$\begin{aligned}\text{Tip Top } &= 50\%A + 30\%B + 20\%C \\ \text{No Name } &= 20\%A + 40\%B + 40\%C\end{aligned}$$

She can sell all that she produces at market prices of $960 per tonne for *Tip Top* and $710 per tonne for *No Name*.

One week the trader is given the option of buying up to 80 tonnes of grade *A* at $800 per tonne, up to 100 tonnes of grade *B* at $600 per tonne and up to 120 tonnes of grade *C* at $400/tonne. Calculate the profit per tonne of each blend of tea. How much of each blend should the trader sell to maximize her profit and what is the maximum profit?
Solve Question 2 by hand using the simplex method

3. A farming company owns two farms, which differ in the growing of crops and their yields. Each farm has 25 acres available for cropping and a total of at least 11,000 bushels of wheat and 7000 bushels of corn must be grown.
Farm *A* yields 400 bushels of wheat per acre at a cost of $100 per acre and 600 bushels of corn per acre at a cost of $120 per acre. Farm *B* yields 300 bushels of wheat per acre at a cost of $90 per acre and 550 bushels of corn per acre at a cost of $110 per acre.

(a)How many acres of each crop should be grown on each farm to minimise the total cost?
(b) What is the minimum cost?
Solve Question 3 using the 2-phase simplex method with Matlab

Q4 2070 only — Formulate *by hand* the dual problem of the primal in Questions 1, 2 and 3. For question 1 only, solve the dual problem using Matlab and compare your answers with the answers to the primal problem.

Q5 2970 only (a) Taper pins (tools used for enlarging holes usually for mechanical purposes) are processed in a factory on two lathes, A and B, and also on a grinder G. The taper pins come in four sizes, identified as #1, #2, #4, and #7. The following table exhibits the processing time in hours per load, required on each machine, for each of the four types of pins. It also lists the net profit per load for each type and the maximum time available on each of the machines, weekly.

Pin Type	Lathe A	Lathe B	Grinder	Profit
#1	10	6	4	$9
#2	1	4	6	$5
#4	5	6	18	$8
#7	2	2	2	$6
Maximum Times	50	36	72	

(i) How many loads of each size taper pin should the factory produce in order to realize a maximum profit for the week's operation? How many iterations through the simplex algorithm are needed to reach the optimal solution?

(ii) Modify the simplex method in the following way. Consider each possible choice of entering variable and its corresponding exiting variable after the initial step. Show that the greatest change in Z occurs for only one of these choices.

(iii) Find the choice of variables leaving the basis and entering the basis which achieves the optimal solution in the minimal number of iteration steps. What is this minimal number?

Solve Question 5(a) by hand

(b) Suppose a standard LP problem is formulated as

$$\text{Maximize } Z = c^T x$$
$$\text{subject to } Ax \le b$$
$$\text{with } x \ge 0$$

where A is an $m \times n$ matrix and c and b are positive vectors. Start with the initialization step, and assume x_{i+n}, $1 \le i \le m$ are slack variables. Suppose in one iteration step, x_q enters the basis and x_p leaves.

(i) What are the new values of x_p, x_q and Z after this iteration?

(ii) What is the criterion for reaching an optimal feasible solution in one step?

(iii) Can an optimal solution be reached in one step for the case when $c_i = \gamma,\ \gamma > 0,\ b_i > 0,\ A_{ij} = ijb_i,$ where $1 \le i \le m,\ 1 \le j \le n$?
Find the specific variables entering and leaving the basis and the optimal value of Z in this case.