Put option  \[ S_2 = 16 \]

- \( S_1 = 8 \)
- \( S_0 = 4 \)
- \( S_{1.5} = 2 \)
- \( S_2 = 1 \)

European Put option holder receives \( (K - S_2)^+ \) at time \( T = 2 \)

\[ V_2 = (K - S_2)^+ \] \[ V_2 (uu) = 0 \] \[ V_2 (ud) = 1 \] \[ V_2 (dd) = 4 \]

American put option: You can exercise at any time \( t = 0, 1, 2 \)

1) Price of the contract at time 0
2) When to exercise?
A02

\[ V_2(s) = (5 - s)^+ \]

At \( t = 1 \) two possibilities: take \((5 - s)^+\) as current value of the stock.

(b) Continue. What to expect if I continue:

\[
\frac{1}{1+r} \mathbb{E} \left[ V_2(S_2) \mid S_1 = s \right]
\]

\[
= \frac{1}{1+r} \left[ \frac{1}{2} V_2(2s) + \frac{1}{2} V_2\left(\frac{s}{2}\right) \right]
\]

\[ V_1(s) = \max \left\{ (5 - s)^+, \frac{2}{5} \left[ V_2(2s) + \frac{1}{2} V_2\left(\frac{s}{2}\right) \right] \right\} \]

\[ V_1(8) = \max \left\{ (5 - 8)^+, \frac{2}{5} (0 + 1)^2 \right\} = 4 \]

\[ V_1(2) = \max \left\{ (5 - 8)^+, \frac{2}{5} (1 + 4)^2 \right\} = 3 \]

\[ V_0(4) = \max \left\{ (5 - 4)^+, \frac{2}{5} (4.4 + 3)^2 \right\} = 1.36 \]

Fair price = 1.36

When to exercise:

At \( t = 1 \) stop if \((5 - s)^+ > \frac{1}{1+r} \mathbb{E} \left[ V_2 \mid S_1 = s \right] \)
\( s = 8 \)
\[(s-8)^+ = 0 \]
\( \text{continue} \)
\[ P\left(\mathcal{A}_2 | S_1\right) = \frac{2}{5} \left(0 + 8\right) \cdot 9 \]
\( s = 2 \)
\[(s-2)^+ = 3 \]
\[ P\left(\mathcal{U}_2 | S_1\right) = 2 \]
\[ \text{stop} \]
\[ \frac{1.36}{6.8 \cdot 5} \]
\( t = 0 \)
\[(s-4)^+ = 1 \]
\[ \text{optimal stopping rule: stop and exercise if} \]
\[ (s-s)^+ \rightarrow \frac{1}{1+t} P\left(\mathcal{U}_2 | S_1\right) \]