Using the 2-phase simplex algorithm with the MATLAB routines:

Procedure:
1. Check: \( \max \)
   \[
   \text{rh} s \geq 0 \\
   \text{variables} \geq 0
   \]

2. Introduce any surplus variables: formulate problem in terms of decision & surplus variables

3. Matlab introduces any slack or artificial variables.

4. You introduce W including all variables - this tells Matlab which variables are artificial.
§ 2.6 The Dual Problem

Consider the given problem

\[ \text{Max } z = 3x_1 + 5x_2 \]
\[ \text{s.t. } \begin{align*}
    x_1 & \leq 4 \\
    2x_2 & \leq 12 \\
    3x_1 + 4x_2 & \leq 16 \\
\end{align*} \]

with \( x_1 \geq 0, x_2 \geq 0 \).

We take linear combinations of the constraint equations to try to upper bound \( z \).

Let \( y_1, y_2, y_3 \geq 0 \). Multiply constraint 1 by \( y_1 \),

\[ \begin{align*}
    y_1 x_1 + 2y_2 x_2 + y_3 (3x_1 + 2x_2) & \leq 4y_1 + 12y_2 + 18y_3 \\
\end{align*} \]

E.g. \( y_1 = 3, y_2 = \frac{5}{2}, y_3 = 0 \) \( \Rightarrow \)

\[ 3x_1 + 5x_2 \leq 3 \times 4 + \frac{5}{2} \times 12 = 42. \]

But \( \text{WHS} = 2 \), so \( 42 \geq 2 \), i.e. 42

is an upper bound on \( z \Rightarrow z^* \leq 42 \).
In general,

\[(y_1 + 3y_3)x_1 + (2y_2 + 2y_3)x_2 \leq 4y_1 + 12y_2 + 18y_3\]

To get \(z = 3x_1 + 5x_2 \leq (y_1 + 3y_3)x_1 + (2y_2 + 2y_3)x_2\)

we must have

\[y_1 + 3y_3 \geq 3 \quad x_1 \geq 0\]
\[2y_2 + 2y_3 \geq 5 \quad x_2 \geq 0\]

Then we get

\[z \leq v = 4y_1 + 12y_2 + 18y_3\]

Define the dual LP problem:

\[
\text{Min } v = 4y_1 + 12y_2 + 18y_3 \\
\text{s.t. } y_1 + 3y_3 \geq 3 \\
2y_2 + 2y_3 \geq 5
\]

with \(y_1, y_2, y_3 \geq 0\).

**Primal**

- \(c^T = [3, 5]\)
- \(b^T = [4, 12, 18]\)
- \(A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 2 \end{bmatrix}\)

**Dual**

- \([4; 12; 18] = b^T\)
- \([3; 5] = c^T\)
- \([1 \ 0 \ 3] = A^T\)
Generally, the primal problem

\[
\begin{align*}
\text{Max } z &= c^T x \\
\text{s.t. } A x &\leq b \\
\text{with } x &> 0 \\
\end{align*}
\]

has dual problem

\[
\begin{align*}
\text{Min } v &= b^T y \\
\text{s.t. } A^T y &\geq c \\
\text{with } y &> 0.
\end{align*}
\]

There is no restriction on \( b \geq 0 \).
Thus any \( \geq \) constraint in the primal can be converted to a \( \leq \) constraint. An equality constraint \( a^T x = b \) can be written as \( a^T x \geq b \) and \( a^T x \leq b \).

The condition \( b \geq 0 \) can be preserved but the formulation of the dual is more complicated.
The dual problem has theoretical use. It also has some practical uses:

1. The number of variables & constraints are interchanged between primal & dual. If the primal has 2 constraints, the dual has 2 decision variables & can be solved graphically.

2. The primal problem may require the 2-phase method for solution, whereas the dual may not.

3. Since the convergence takes about twice the number of constraints, the dual problem may converge faster in the simplex algorithm (one or 2-phase).

These all presuppose that the solutions of the primal & dual are related. This is the case: $x^* = y^*$.
Eq

Min \quad Z = \quad 3x_1 + 2x_2
\text{ s.t. } \quad 8x_1 + 3x_2 \geq 24
\quad 5x_1 + 6x_2 \geq 30
\quad 2x_1 + 9x_2 \geq 18
\quad \text{with } \quad x_1, x_2 \geq 0.

The dual problem is

Max \quad v = \quad 24y_1 + 30y_2 + 18y_3
\text{ s.t. } \quad 8y_1 + 5y_2 + 2y_3 \leq 3
\quad 3y_1 + 6y_2 + 9y_3 \leq 2
\quad \text{with } \quad y_1, y_2, y_3 \geq 0.