Lab 4 Solutions

```
% Question 2
% See Tutorial 1 for formulation.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Min z = 8x1 + 12x2
% subject to:  x1 + x2 >=  7
%             2x1 + x2 >= 10
% with         x1,x2 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Maximise: zhat = -z.
% Decision variables: x1,x2.
% Introduce surplus variables: x3,x4.
% Problem becomes
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Max zhat = - 8x1 - 12x2 + 0x3 + 0x4
% subject to:  x1 + x2 - x3      =  7
%             2x1 + x2      - x4 = 10
% with        x1,x2,x3,x4 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Enter problem in terms of decision variables and surplus variables.
% Matlab will introduce artificial variables x5,x6.
zhat=[-8,-12,0,0];
A=[1,1,-1,0;2,1,0,-1];
b=[7;10];
w=[0,0,0,0,-1,-1];
% w contains a 0 for each decision, surplus and slack variable,
% and a -1 for each artifical variable. First the decision and
% surplus variables are treated in the order in which they are
% labelled. Then the slack and artifical variables are treated
% in the order of their constraints in A.
t0=tableau2(A,b,zhat,w);
```

W	Z	x1	x2	x3	x4	$\overline{x5}$	$\overline{x6}$	RHS
1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
0.00	1.00	8.00	12.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	7.00
0.00	0.00	2.00	1.00	0.00	-1.00	0.00	1.00	10.00

```
t0(1,:)=t0(1,:)-t0(3,:)-t0(4,:);
displaytableau2(t0)
```

W	Z	x1	x2	x3	x4	$\overline{x5}$	$\overline{x6}$	RHS
1.00	0.00	-3.00	-2.00	1.00	1.00	0.00	0.00	-17.00
0.00	1.00	8.00	12.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	7.00
0.00	0.00	2.00	1.00	0.00	-1.00	0.00	1.00	10.00

```
ratios2(3,t0);
```

Ratios of RHS to column 3:

W	Z	x1	x2	x3	x4	$\overline{x5}$	$\overline{x6}$	RHS	Ratio
1.00	0.00	-3.00	-2.00	1.00	1.00	0.00	0.00	-17.00	-
0.00	1.00	8.00	12.00	0.00	0.00	0.00	0.00	0.00	-
0.00	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	7.00	7.00
0.00	0.00	2.00	1.00	0.00	-1.00	0.00	1.00	10.00	5.00

```
t1=pivot2(4,3,t0);
```

W	Z	x1	x2	x3	x4	$\overline{x5}$	$\overline{x6}$	RHS
1.00	0.00	0.00	-0.50	1.00	-0.50	0.00	1.50	-2.00
0.00	1.00	0.00	8.00	0.00	4.00	0.00	-4.00	-40.00
0.00	0.00	0.00	0.50	-1.00	0.50	1.00	-0.50	2.00
0.00	0.00	1.00	0.50	0.00	-0.50	0.00	0.50	5.00

```
-----------------------------------------------------------------------------
ratios2(4,t1);

Ratios of RHS to column  4:

   W        Z    |   x1      x2      x3      x4    |   x5      x6    |     RHS    |  Ratio
-------------------------------------------------------------------------------------------
  1.00    0.00   |   0.00   -0.50    1.00   -0.50  |   0.00    1.50  |    -2.00   |    -
  0.00    1.00   |   0.00    8.00    0.00    4.00  |   0.00   -4.00  |   -40.00   |    -
  0.00    0.00   |   0.00    0.50   -1.00    0.50  |   1.00   -0.50  |     2.00   |   4.00
  0.00    0.00   |   1.00    0.50    0.00   -0.50  |   0.00    0.50  |     5.00   |  10.00
-------------------------------------------------------------------------------------------
t2=pivot2(3,4,t1);
   W        Z    |   x1      x2      x3      x4    |   x5      x6    |     RHS
-----------------------------------------------------------------------------
  1.00    0.00   |   0.00    0.00    0.00    0.00  |   1.00    1.00  |     0.00
  0.00    1.00   |   0.00    0.00   16.00   -4.00  | -16.00    4.00  |   -72.00
  0.00    0.00   |   0.00    1.00   -2.00    1.00  |   2.00   -1.00  |     4.00
  0.00    0.00   |   1.00    0.00    1.00   -1.00  |  -1.00    1.00  |     3.00
-----------------------------------------------------------------------------
% Tableau t2 is optimal for w and w*=0. Phase 1 successful.
% Begin phase 2.
t3=phase2(t2);

   Z    |   x1      x2      x3      x4    |     RHS
-------------------------------------------------------
  1.00  |   0.00    0.00   16.00   -4.00  |   -72.00
  0.00  |   0.00    1.00   -2.00    1.00  |     4.00
  0.00  |   1.00    0.00    1.00   -1.00  |     3.00
-------------------------------------------------------
ratios(5,t3);

Ratios of RHS to column  5:

   Z    |   x1      x2      x3      x4    |     RHS    |  Ratio
-------------------------------------------------------------------
  1.00  |   0.00    0.00   16.00   -4.00  |   -72.00   |    -
  0.00  |   0.00    1.00   -2.00    1.00  |     4.00   |   4.00
  0.00  |   1.00    0.00    1.00   -1.00  |     3.00   |    -
-------------------------------------------------------------------
t4=pivot(2,5,t3);

   Z    |   x1      x2      x3      x4    |     RHS
-------------------------------------------------------
  1.00  |   0.00    4.00    8.00    0.00  |   -56.00
  0.00  |   0.00    1.00   -2.00    1.00  |     4.00
  0.00  |   1.00    1.00   -1.00    0.00  |     7.00
-------------------------------------------------------
% Tableau t4 is optimal.
% Optimal solution: x1*=7,x2*=0,x3*=0,x4*=4,z*=-zhat*=56.
%------------------------------------------------------------------
% Question 3
% See Tutorial 1 for formulation.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Min z = 10x1 + 12x2  (units=k$)
% subject to:   8x1 +   8x2 <=   48
%             100x1 + 200x2  = 1000
% with          x1,x2 >= 0.
%
% A more general formulation would be to
% replace the second constraint by:
% 100x1 + 200x2 >= 1000.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Maximise: zhat = -z.
% Decision variables: x1,x2.
% Problem becomes
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Max zhat = - 10x1 - 12x2  (units=$k)
```

```
% subject to:   8x1 +   8x2 <=   48
%             100x1 + 200x2  = 1000
% with          x1,x2 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Enter problem in terms of decision variables (no surplus variables).
% Matlab will introduce slack variable x3 and artificial variable x4.
zhat=[-10,-12];
A=[8,8;100,200];
b=[48;1000];
w=[0,0,0,-1];
t0=tableau2(A,b,zhat,w);

   W       Z    |   x1      x2    |   x3      x4    |      RHS
--------------------------------------------------------------
  1.00   0.00   |   0.00    0.00  |  0.00    1.00   |      0.00
  0.00   1.00   |  10.00   12.00  |  0.00    0.00   |      0.00
  0.00   0.00   |   8.00    8.00  |  1.00    0.00   |     48.00
  0.00   0.00   | 100.00 200.00   |  0.00    1.00   |   1000.00
--------------------------------------------------------------
t0(1,:)=t0(1,:)-t0(4,:);
displaytableau2(t0)

   W       Z    |   x1      x2    |   x3      x4    |      RHS
--------------------------------------------------------------
  1.00   0.00   | -100.00 -200.00 |   0.00    0.00  |  -1000.00
  0.00   1.00   |  10.00   12.00  |  0.00    0.00   |      0.00
  0.00   0.00   |   8.00    8.00  |  1.00    0.00   |     48.00
  0.00   0.00   | 100.00 200.00   |  0.00    1.00   |   1000.00
--------------------------------------------------------------
ratios2(4,t0);

Ratios of RHS to column  4:

   W       Z    |   x1      x2    |   x3      x4    |      RHS    | Ratio
------------------------------------------------------------------------
  1.00   0.00   | -100.00 -200.00 |   0.00    0.00  |  -1000.00   |    -
  0.00   1.00   |  10.00   12.00  |  0.00    0.00   |      0.00   |    -
  0.00   0.00   |   8.00    8.00  |  1.00    0.00   |     48.00   |   6.00
  0.00   0.00   | 100.00 200.00   |  0.00    1.00   |   1000.00   |   5.00
------------------------------------------------------------------------
t1=pivot2(4,4,t0);

   W       Z    |   x1      x2    |   x3      x4    |      RHS
--------------------------------------------------------------
  1.00   0.00   |   0.00    0.00  |  0.00    1.00   |      0.00
  0.00   1.00   |   4.00    0.00  |  0.00   -0.06   |    -60.00
  0.00   0.00   |   4.00    0.00  |  1.00   -0.04   |      8.00
  0.00   0.00   |   0.50    1.00  |  0.00    0.01   |      5.00
--------------------------------------------------------------
% Tableau t2 is optimal for w and w*=0. Phase 1 successful.
% Begin phase 2.
t2=phase2(t1);

   Z    |   x1      x2      x3    |     RHS
---------------------------------------------
  1.00  |  4.00    0.00    0.00   |   -60.00
  0.00  |  4.00    0.00    1.00   |     8.00
  0.00  |  0.50    1.00    0.00   |     5.00
---------------------------------------------
% Tableau t2 is optimal.
% Optimal solution: x1*=0,x2*=5,x3*=8,z*=-zhat*=$60k.
%-------------------------------------------------------------
% Question 4
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Max z = - 2x1 - 6x2 + 7x3 - 2x4 - 4x5
% subject to:  4x1 - 3x2 +  8x3 -  x4        = 12
%                  -  x2 + 12x3 - 3x4 + 4x5 = 20
% with          x1,x2,x3,x4,x5 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Decision variables: x1,x2,x3,x4,x5.
% Enter problem in terms of decision variables.
% Matlab will introduce artificial variables x6,x7.
z=[-2,-6,7,-2,-4];
A=[4,-3,8,-1,0;0,-1,12,-3,4];
b=[12;20];
w=[0,0,0,0,0,-1,-1];
t0=tableau2(A,b,z,w);
```

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS
1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
0.00	1.00	2.00	6.00	-7.00	2.00	4.00	0.00	0.00	0.00
0.00	0.00	4.00	-3.00	8.00	-1.00	0.00	1.00	0.00	12.00
0.00	0.00	0.00	-1.00	12.00	-3.00	4.00	0.00	1.00	20.00

```
t0(1,:)=t0(1,:)-t0(3,:)-t0(4,:);
displaytableau2(t0)
```

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS
1.00	0.00	-4.00	4.00	-20.00	4.00	-4.00	0.00	0.00	-32.00
0.00	1.00	2.00	6.00	-7.00	2.00	4.00	0.00	0.00	0.00
0.00	0.00	4.00	-3.00	8.00	-1.00	0.00	1.00	0.00	12.00
0.00	0.00	0.00	-1.00	12.00	-3.00	4.00	0.00	1.00	20.00

```
ratios2(5,t0);
```

Ratios of RHS to column 5:

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS	Ratio
1.00	0.00	-4.00	4.00	-20.00	4.00	-4.00	0.00	0.00	-32.00	-
0.00	1.00	2.00	6.00	-7.00	2.00	4.00	0.00	0.00	0.00	-
0.00	0.00	4.00	-3.00	8.00	-1.00	0.00	1.00	0.00	12.00	1.50
0.00	0.00	0.00	-1.00	12.00	-3.00	4.00	0.00	1.00	20.00	1.67

```
t1=pivot2(3,5,t0);
```

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS
1.00	0.00	6.00	-3.50	0.00	1.50	-4.00	2.50	0.00	-2.00
0.00	1.00	5.50	3.38	0.00	1.13	4.00	0.88	0.00	10.50
0.00	0.00	0.50	-0.38	1.00	-0.13	0.00	0.13	0.00	1.50
0.00	0.00	-6.00	3.50	0.00	-1.50	4.00	-1.50	1.00	2.00

```
ratios2(7,t1);
```

Ratios of RHS to column 7:

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS	Ratio
1.00	0.00	6.00	-3.50	0.00	1.50	-4.00	2.50	0.00	-2.00	-
0.00	1.00	5.50	3.38	0.00	1.13	4.00	0.88	0.00	10.50	-
0.00	0.00	0.50	-0.38	1.00	-0.13	0.00	0.13	0.00	1.50	-
0.00	0.00	-6.00	3.50	0.00	-1.50	4.00	-1.50	1.00	2.00	0.50

```
t2=pivot2(4,7,t1);
```

W	Z	x1	x2	x3	x4	x5	$\bar{x}6$	$\bar{x}7$	RHS
1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
0.00	1.00	11.50	-0.13	0.00	2.63	0.00	2.38	-1.00	8.50
0.00	0.00	0.50	-0.38	1.00	-0.13	0.00	0.13	0.00	1.50
0.00	0.00	-1.50	0.88	0.00	-0.38	1.00	-0.38	0.25	0.50

```
% Tableau t2 is optimal for w and w*=0. Phase 1 successful.
% Begin phase 2.
t3=phase2(t2);
```

```
   Z   |  x1      x2      x3      x4      x5    |       RHS
-------------------------------------------------------------
  1.00 | 11.50  -0.13   0.00    2.63    0.00  |        8.50
  0.00 |  0.50  -0.38   1.00   -0.13    0.00  |        1.50
  0.00 | -1.50   0.88   0.00   -0.38    1.00  |        0.50
-------------------------------------------------------------
ratios(3,t3);

Ratios of RHS to column  3:

   Z   |  x1      x2      x3      x4      x5    |       RHS   | Ratio
---------------------------------------------------------------------------
  1.00 | 11.50  -0.13   0.00    2.63    0.00  |        8.50  |      -
  0.00 |  0.50  -0.38   1.00   -0.13    0.00  |        1.50  |      -
  0.00 | -1.50   0.88   0.00   -0.38    1.00  |        0.50  |     0.57
---------------------------------------------------------------------------
t4=pivot(3,3,t3);

   Z   |  x1      x2      x3      x4      x5    |       RHS
-------------------------------------------------------------
  1.00 | 11.29   0.00   0.00    2.57    0.14  |        8.57
  0.00 | -0.14   0.00   1.00   -0.29    0.43  |        1.71
  0.00 | -1.71   1.00   0.00   -0.43    1.14  |        0.57
-------------------------------------------------------------
% Tableau t4 is optimal.
% Optimal solution: x1*=0,x2*=0.57,x3*=1.71,x4*=0,x5*=0,z*=8.57.
%--------------------------------------------------------------------
% Question 5
% (a)
%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Min z = 8x1 + 12x2
% subject to:  x1 + x2 >=  7
%             2x1 + x2 >= 10
% with         x1,x2 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% There is no restriction on the sign
% of b, so make all constraints
% less-than-or-equal-to A*x <= b:
% Min z = 8x1 + 12x2
% subject to: - x1 - x2 <=  -7
%             -2x1 - x2 <= -10
% with         x1,x2 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
z=[8 12];
A=[-1 -1 ; -2 -1];
b=[-7;-10];
lb=[0 0];  % x1,x2 >= 0.
x=linprog(z,A,b,[],[],lb,[])
Optimization terminated successfully.
x =
    7.0000
    0.0000
z*x
ans =
   56.0000
%Optimal solution:
%x1*=7, x2*=0, z*=56.
%-----------------------------------------------------------------
% (b)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Max z = - 2x1 - 6x2 + 7x3 - 2x4 - 4x5
% subject to:  4x1 - 3x2 +  8x3 -  x4        = 12
%                  -  x2 + 12x3 - 3x4 + 4x5 = 20
% with           x1,x2,x3,x4,x5 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem must be minimisation form, Max -z.
% Enter less-than-or-equal-to constraints A*x <= b
% and equal-to constraints Aeq*x = beq separately:
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
z=[-2,-6,7,-2,-4];
Aeq=[4,-3,8,-1,0;0,-1,12,-3,4];
beq=[12;20];
lb=[0 0 0 0 0];
x=linprog(-z,[],[],Aeq,beq,lb,[])
Optimization terminated successfully.
x =
    0.0000
    0.5714
    1.7143
    0.0000
    0.0000
z*x
ans =
    8.5714
% Optimal solution: x1*=0,x2*=0.57,x3*=1.71,x4*=0,x5*=0,z*=8.57.
%-----------------------------------------------------------------
% Question 6
% See Tutorial 1 for formulation.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Min z = x1 + 1.5x2
% subject to: 4x1 + 10x2 >= 100
%             4x1 +  2x2 >=  60
% with         x1,x2 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Introduce surplus variables: x3,x4. Problem becomes:
%
% Max zhat = - x1 - 1.5x2
% subject to: 4x1 + 10x2 - x3      = 100
%             4x1 +  2x2      - x4 =  60
% with         x1,x2,x3,x4 >= 0.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Introduce artificial variables x5,x6 and a new objective function
% Max Z = zhat + M*w = zhat - Mx5 - Mx6
% If M is large enough the 1 phase simplex method will force the
% artificial variables x5,x6 with cost coefficients M to go to zero
% first. The difficulty with the numerical big-M method, as in Matlab,
% is that M must be given a large numerical value and it is often
% not clear how big ``big'' should be. By hand M can be left as a symbol,
% which is bigger than any other number. We choose M=100 and hope it works.
% Enter A, b in terms of decision and surplus variables:
A=[4,10,-1,0;4,2,0,-1];
b=[100;60];
% Enter zhat in terms of all variables, including artificial:
M=100;
zhat=[-1,-1.5,0,0,-M,-M];
t0=tableau(A,b,zhat);
Big M method:

   Z    |   x1      x2      x3      x4      x5      x6    |      RHS
-----------------------------------------------------------------
  1.00  |   1.00    1.50    0.00    0.00  100.00  100.00  |      0.00
  0.00  |   4.00   10.00   -1.00    0.00    1.00    0.00  |    100.00
  0.00  |   4.00    2.00    0.00   -1.00    0.00    1.00  |     60.00
-----------------------------------------------------------------
% Express objective in terms of non-basic variables:
t0(1,:)=t0(1,:)-M*(t0(2,:)+t0(3,:));
displaytableau(t0);

   Z    |   x1      x2      x3      x4      x5      x6    |      RHS
-----------------------------------------------------------------
  1.00  | -799.00 -1198.50 100.00 100.00    0.00    0.00  | -16000.00
  0.00  |   4.00   10.00   -1.00    0.00    1.00    0.00  |    100.00
  0.00  |   4.00    2.00    0.00   -1.00    0.00    1.00  |     60.00
-----------------------------------------------------------------
ratios(3,t0);

Ratios of RHS to column  3:
```

```
    Z    |   x1      x2      x3      x4      x5      x6    |      RHS     |  Ratio
-------------------------------------------------------------------------------------
  1.00   | -799.00 -1198.50 100.00 100.00    0.00    0.00 | -16000.00 |        -
  0.00   |   4.00   10.00   -1.00    0.00    1.00    0.00 |    100.00 |     10.00
  0.00   |   4.00    2.00    0.00   -1.00    0.00    1.00 |     60.00 |     30.00
-------------------------------------------------------------------------------------
t1=pivot(2,3,t0);

    Z    |   x1      x2      x3      x4      x5      x6    |      RHS
----------------------------------------------------------------------
  1.00   | -319.60    0.00  -19.85  100.00  119.85    0.00 |  -4015.00
  0.00   |   0.40    1.00   -0.10    0.00    0.10    0.00 |     10.00
  0.00   |   3.20    0.00    0.20   -1.00   -0.20    1.00 |     40.00
----------------------------------------------------------------------
ratios(2,t1);

Ratios of RHS to column  2:

    Z    |   x1      x2      x3      x4      x5      x6    |      RHS     |  Ratio
-------------------------------------------------------------------------------------
  1.00   | -319.60    0.00  -19.85  100.00  119.85    0.00 |  -4015.00 |        -
  0.00   |   0.40    1.00   -0.10    0.00    0.10    0.00 |     10.00 |     25.00
  0.00   |   3.20    0.00    0.20   -1.00   -0.20    1.00 |     40.00 |     12.50
-------------------------------------------------------------------------------------
t2=pivot(3,2,t1);

    Z    |   x1      x2      x3      x4      x5      x6    |      RHS
----------------------------------------------------------------------
  1.00   |   0.00    0.00    0.12    0.13   99.88   99.87 |    -20.00
  0.00   |   0.00    1.00   -0.13    0.13    0.13   -0.13 |      5.00
  0.00   |   1.00    0.00    0.06   -0.31   -0.06    0.31 |     12.50
----------------------------------------------------------------------
% Tableau t2 is optimal with the artifical variables x5*=x6*=0.
% Optimal solution: x1*=12.5,x2*=5,x3*=0,x4*=0,z*=-zhat*=20.
```