Computer Lab 9–12

*For the four weeks beginning Monday 8th October.*

*Advanced questions are marked A.*

**Project**

This project is worth 10 marks and is due 5:00pm on Thursday 1st November

This is mostly a computational project so you must submit all computer programs with your project formulations, descriptions and outputs. Assessment will be based on: accuracy, programming and presentation. Normal level students do Questions 1 to 4 only; advanced level students do all the Questions.

**The Scenario:** Pythagoras Jones has just inherited $1,000,000 and wishes to invest this sum in the five funds listed below.

<table>
<thead>
<tr>
<th>Fund</th>
<th>Name</th>
<th>Code</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Aussie Growth Limits</td>
<td>(AGL)</td>
<td>.065</td>
<td>.035</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Wealthy People Leverage</td>
<td>(WPL)</td>
<td>.080</td>
<td>.025</td>
</tr>
<tr>
<td>$P_3$</td>
<td>Christmas Stocking Revenues</td>
<td>(CSR)</td>
<td>.105</td>
<td>.030</td>
</tr>
<tr>
<td>$P_4$</td>
<td>Best Haven Protection</td>
<td>(BHP)</td>
<td>.115</td>
<td>.040</td>
</tr>
<tr>
<td>$P_5$</td>
<td>Cash Benefit Assets</td>
<td>(CBA)</td>
<td>.135</td>
<td>.050</td>
</tr>
</tbody>
</table>

Funds $P_1$, $P_3$ have a negative correlation coefficient $-0.3$ and funds $P_2$, $P_5$ a positive correlation coefficient $+0.6$. All other pairs of funds are uncorrelated. There are no restrictions on short selling and Pythagoras has a risk aversion parameter measured to be $t = 0.005$ units.

1. Determine which investors short sell in this market and which funds they short sell. Are there any funds which no-one will short sell?
2. **Pythagoras’s Optimal Portfolio**: Carry out the following computational tasks for Pythagoras’s optimal portfolio $P^*$. 

(a) Obtain the dollar investment in each of the five funds and obtain the corresponding expected return and risk of $P^*$.

(b) Obtain the $\mu\sigma$-plane graphical representation and include (all on the same graph):

(i) The five investment funds.

(ii) The minimum variance and efficient frontiers. Use a $t$-range $|t| \leq 0.06$ for your display.

(iii) A plot of (about) $10^3$ random feasible portfolios satisfying $|x_i| \leq 2$ (for each of the 5 funds) and $\sigma_i \leq 0.1$ for $i = 1, \ldots, 1000$.

(iv) Pythagoras’s indifference curve and optimal portfolio $P^*$.

(c) Repeat the $\mu\sigma$-plane graph of Part (b) but plot $10^4$ random feasible portfolios satisfying $x_i \geq 0$ (for each of the 5 funds), i.e. no short-selling.

3. **Adding a Riskless Cash Fund**: Suppose now that a riskless cash fund $P_0$ is also available to invest in. The risk free rate is 0.05 for both lending and borrowing. Obtain Pythagoras’s new allocation of his inheritance to the (now) six funds. State clearly Pythagoras’s investment in the riskless cash fund and describe in detail the tangency portfolio.

4. **The Capital Market Line**: Make a new $\mu\sigma$-plane graph showing the riskless cash fund, tangency portfolio, Pythagoras’s new optimal portfolio and the Capital Market Line relative to the risky efficient frontier. If the five original funds have a net worth of $350$ million, estimate (to the nearest $0.1$ million) the total value of each fund.

5. **The Security Market Line**: Compute the beta’s of all relevant funds and assets in this project and clearly display them on the Security Market Line. Comment on the result.