1.

The final simplex tableau is

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	2	0	1	0	3	6
0	−1	0	1	1	−1	2
0	1	1	1	0	1	2

(a) The columns $\begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}$ indicate the basic variables are x_2, x_4. The other variables, x_1, x_3, x_5, are non-basic.

(b) Row 1 gives the equation, $Z + 2x_1 + x_3 + 3x_5 = 6$. Since the non-basic variables are 0, $x_1 = x_3 = x_5 = 0$ and $Z = 6$. The tableau is <u>optimal</u> since there are no negative coefficients in row 1. $\therefore Z^* = 6$.

(c) $x_1^* = 0,\ x_2^* = 2,\ x_3^* = 0,\ x_4^* = 2,\ x_5^* = 0$.
The solution vector is $(0, 2, 0)$.

(d) Rows 2, 3 give the constraint equations,

$$① \quad -x_1 \qquad + x_3 + x_4 - x_5 = 2$$
$$② \quad x_1 + x_2 + x_3 \qquad + x_5 = 2$$

In the initial tableau the constraints must be of the form

$$③ \quad \times x_1 + \times x_2 + \times x_3 + x_4 \qquad = \times$$
$$④ \quad \times x_1 + \times x_2 + \times x_3 \qquad + x_5 = \times$$

where the $\times$'s represent unknown values. Constraint ② is already of the form ④. ① + ② gives

$$⑤ \quad x_2 + 2x_3 + x_4 = 4$$

which is of the required form ③. Removing the slack variables x_4, x_5 from ② and ⑤ gives

(iv) $x_1 + x_2 + x_3 \leqslant 2$, (vii) $x_2 + 2x_3 \leqslant 4$.

2. Check problem is in standard form:

1) Max 2) RHS's ≥ 0 3) constraints $\leq$ 4) variables ≥ 0

Introduce slack variables x_3, x_4:

$$
\begin{aligned}
2x_1 + 3x_2 + x_3 \qquad &= 12 \\
-3x_1 + 2x_2 \qquad + x_4 &= 6
\end{aligned}
$$

	z	x_1	x_2	x_3	x_4	RHS	Ratio	
	1	-4	-3	0	0	0	—	x_1 enters
	0	$\boxed{2}$	3	1	0	12	6	← x_3 leaves
	0	-3	2	0	1	6	—	
$R_1 := R_1 + 2R_2$	1	0	3	2	0	24		Optimal
$R_2 := R_2/2$	0	1	$3/2$	$1/2$	0	6		
$R_3 := R_3 + \frac{3}{2}R_2$	0	0	$13/2$	$3/2$	1	24		

Tableau is optimal because all coefficients in row 1 are ≥ 0. The optimal solution is

$$z^* = 24, \quad x_1^* = 6, \quad x_2^* = 0, \quad x_3^* = 0, \quad x_4^* = 24.$$

3.

Check problem is in standard form.

Introduce slack variables x_4, x_5 :

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 \quad &= 12 \\ 3x_1 - x_2 + 3x_3 \quad + x_5 &= 8 \end{aligned}$$

	z	x_1	x_2	x_3	x_4	x_5	RHS	Ratio	
	1	3	-2	-3	0	0	0	—	x_3 enters
	0	2	1	1	1	0	12	12	
	0	3	-1	$\boxed{3}$	0	1	8	$8/3$	← x_5 leaves
$R_1 := R_1 + R_3$	1	6	-3	0	0	1	8	—	x_2 enters
$R_2 := R_2 - \frac{1}{3}R_3$	0	1	$\boxed{4/3}$	0	1	$-1/3$	$28/3$	7	← x_4 leaves
$R_3 := \frac{1}{3}R_3$	0	1	$-1/3$	1	0	$1/3$	$8/3$	—	
	1	$33/4$	0	0	$9/4$	$1/4$	29		Optimal
	0	$3/4$	1	0	$3/4$	$-1/4$	7		
	0	$5/4$	0	1	$1/4$	$1/4$	5		

Tableau is optimal with solution

$$z^* = 29, \quad x_1^* = 0, \quad x_2^* = 7, \quad x_3^* = 5, \quad x_4^* = 0, \quad x_5^* = 0.$$

z	x_1	x_2	x_3	x_4	x_5	RHS	Ratio	
1	-2	-2	0	0	0	0	—	x_1 enters
0	1	2	1	0	0	11	11	
0	$\boxed{2}$	1	0	1	0	10	5	x_4 leaves
0	1	1	0	0	1	6	6	
1	0	-1	0	1	0	10	—	x_2 enters
0	0	$3/2$	1	$-1/2$	0	6	4	
0	1	$1/2$	0	$1/2$	0	5	10	
0	0	$\boxed{1/2}$	0	$-1/2$	1	1	2	x_5 leaves
1	0	0	0	0	2	12		Optimal
0	0	0	1	1	-3	3		
0	1	0	0	1	-1	4		
0	0	1	0	-1	2	2		

$$x_1^* = 4, \quad x_2^* = 2, \quad x_3^* = 3, \quad x_4^* = 0, \quad x_5^* = 0, \quad z^* = 12$$

The modified cost coefficient of the non-basic variable x_4 is zero. $\therefore$ ∞-many solutions.
Let $x_4^* = t$, $x_5^* = 0$. Then

$$x_3^* + x_4^* = 3 \Rightarrow x_3^* = 3-t \geq 0 \Rightarrow t \leq 3$$
$$x_1^* + x_4^* = 4 \Rightarrow x_1^* = 4-t \geq 0 \Rightarrow t \leq 4$$
$$x_2^* - x_4^* = 2 \Rightarrow x_2^* = 2+t \geq 0 \Rightarrow t \geq -2$$
$$x_4^* \geq 0 \Rightarrow t \geq 0.$$

$\therefore$ optimal solutions are

$$x_1^* = 4-t$$
$$x_2^* = 2+t$$
$$x_3^* = 3-t$$
$$x_4^* = t$$
$$x_5^* = 0$$
$$z^* = 12.$$

5 Let x_1 = no. of minutes allotted to comedian

x_2 = " " " " " commercials

$x_3 = 30 - x_1 - x_2 =$ " " " " " band.

Then problem is to

case(a): maximise no. of viewers $Z_1 = 4000x_1 - 1000x_2 + 2000x_3$

$$\text{ie } Z_1 = 1000(2x_1 - 3x_2 + 60)$$

case(b): minimise total cost $Z_2 = 150x_1 + 50x_2 + 100x_3$

$$\text{ie } Z_2 = 50(x_1 - x_2 + 60)$$

Constraints:

The sponsor insists $x_2 \geq 3$

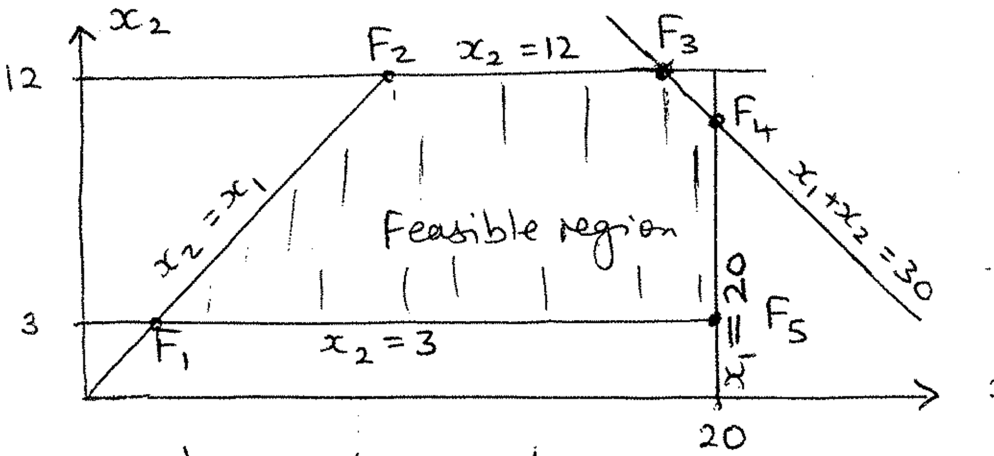
The TV company insists $x_2 \leq 12$

$x_2 \leq x_1$

The comedian insists $x_1 \leq 20$

Also $x_1, x_2 \geq 0$, and $x_3 \geq 0$, $\Rightarrow x_1 + x_2 \leq 30$.

Draw these constraints in the $x_1 - x_2$ plane:



Feasible region is as indicated.

FCPs	Z_1	Z_2
F_1 (3,3)	57,000	3000*
F_2 (12,12)	48,000	3000*
F_3 (18,12)	60,000	3300
F_4 (20,10)	70,000	3500
F_5 (20,3)	91,000*	3850

The table of corner point values shows that for case (a) it is optimal to take $x_1^* = 20$, $x_2^* = 3$, $\Rightarrow x_3^* = 7$, with maximal no. of viewers $Z_1^* = 91,000$. For case (b) the solution is non-unique: the minimum cost is $Z_2^* = \$3000$, for any operating point on the line $F_1 F_2$.

(These results could also be found by looking at the slopes of the objective functions)