Tutorial 3 — Week 4

Tutorial questions are starred. Advanced questions are marked A.

1* Solve by hand using the two-phase method. A hospital wants to design a dinner menu containing two items $M$ and $N$. Each gm of $M$ provides 1 unit of vitamin $A$ and 2 units of vitamin $B$. Each gm of $N$ provides 1 unit of vitamin $A$ and 1 unit of vitamin $B$. The two dishes must provide at least 7 units of vitamin $A$ and at least 10 units of vitamin $B$. If each gm of $M$ costs 8 cents and each gm of $N$ costs 12 cents, how many gm of each item should the hospital serve to minimize its cost?

2* Solve by hand using the two-phase method. Holiday Airline Service wants to fly 1000 members of a travel club to Rome. The airline owns two types of planes, Type $A$ can carry 100 passengers and type $B$ can carry 200 passengers. Type $A$ will cost the airline $10,000 for the trip and type $B$ will cost $12,000 for the trip. If each aeroplane requires eight stewardesses and there are only 48 stewardesses available, how many planes of each type should be used to minimize the airline’s cost for the trip?

3A The following optimisation problems are often used to find a best approximate solution to an inconsistent set of $m$ equations in $n$ unknowns,

$$
\sum_{j=1}^{n} a_{ij}x_j = b_i, \quad i = 1, \ldots, m.
$$

Convert these optimisation problems into equivalent LP problems.

(a) $(L_1$-Approximation) Minimise the sum of the absolute errors,

$$
\sum_{i=1}^{m} \left| b_i - \sum_{j=1}^{n} a_{ij}x_j \right|,
$$

with respect to $x_k, \ k = 1, \ldots, n$. [Hint: Use $n + m$ decision variables.]

(b) $(L_\infty$-Approximation) Minimise the maximum absolute error,

$$
\max \left| b_i - \sum_{j=1}^{n} a_{ij}x_j \right|,
$$

with respect to $x_k, \ k = 1, \ldots, n$. [Hint: Use $n + 1$ decision variables.]

4A Prove for LP problems that the dual of the dual is the primal.