Tutorial 6 — Week 7

Tutorial questions are starred. Advanced questions are marked A.

1* Let $X$ be the sum of the numbers showing when two dice are thrown. Sketch the probability distribution of $X$ and calculate its mean and variance.

2*A The discrete binomial distribution is defined by the probability function

$$b(k; n, p) = \binom{n}{k} p^k q^{n-k}; \quad 0 \leq k \leq n$$

and represents the probability of $k$ successes in $n$ independent trials when $p$ is the probability of a success in a single trial and $q = 1 - p$.

Show that $E[X] = np$ and $\text{Var}(X) = npq$, where $\text{Var}(X)$ denotes the variance of $X$.

3* The yield $Y$ of a zero-coupon bond is assumed to be a continuous random variable, exponentially distributed with p.d.f. given by

$$f(Y) = \frac{1}{\mu} e^{-Y/\mu}; \quad Y \geq 0.$$ 

Find the expected value of the yield.

4* Given that the continuous compound rate $R$ is normally distributed with mean $\mu$ and variance $\sigma^2$, show that the expected discount factor is

$$E[e^{-Rt}] = e^{-(\mu - \sigma^2 t/2)t}.$$