# Tutorial 10 Solutions

(1) a) $S_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = 1428 - 10(11)^2 = 218$

$S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} = 454.2 - 10(11)(3.33) = 87.9$

$\Rightarrow \quad \hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{87.9}{218} = 0.403$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 3.33 - 0.403(11) = -1.105$

$\Rightarrow \quad \boxed{\hat{y} = -1.105 + 0.403x}$.

b) $S_{yy} = \sum_i y_i^2 - n\bar{y}^2 = 149.37 - 10(3.33)^2 = 38.481$

$SS_{Regr.} = \frac{S_{xy}^2}{S_{xx}} = 35.442$

Therefore, the ANOVA table is

| Source | df | SS | MS | F |
|---|---|---|---|---|
| Regr. | 1 | 35.442 | 35.442 | 93.31 |
| Resid. | 8 | 3.039 | 0.38 | |
| Total | 9 | 38.481 | | |

From F-table, $F_{1,8,0.05} = 5.32$

$\Rightarrow$ We reject $H_0: \beta = 0$ @ $\alpha = 0.05$, because $93.31 > 5.32$

c) $\hat{y}_{30} = -1.105 + 0.403(30) = 10.99$

$\Rightarrow$ 95% Prediction interval is $10.99 \pm t_{8,0.025}\sqrt{0.38}\sqrt{\frac{1}{10} + \frac{(30-11)^2}{218}}$

$= 10.99 \pm 2.306\sqrt{0.38}\sqrt{1.756} = 10.99 \pm 1.883$

$= \boxed{(9.12, 12.87)}$.

d) $\hat{y}_{20} = -1.105 + 0.403(20) = 6.96$

PI is $6.96 \pm 2.306\sqrt{0.38}\sqrt{1 + \frac{1}{10} + \frac{(20-11)^2}{218}} = 6.96 \pm 0.98$

$= \boxed{(5.28, 8.68)}$

(2)

a) $\bar{x} = 12$ , $\bar{y} = 23.9$

$\Rightarrow$ $S_{xx} = 1608 - 10(12)^2 = 168$

$S_{xy} = 2950 - 10(12)(23.9) = 82$

$\Rightarrow$ $\hat{\beta} = \frac{82}{168} = 0.488$ , $\hat{\alpha} = 23.9 - 0.488(12) = 18.043$

$\Rightarrow$ $\boxed{\hat{y} = 18.043 + 0.488x}$

b) $H_0: \beta = 0$ vs. $H_1: \beta \neq 0$

T.S. $t = \frac{\sqrt{n-2}\, S_{xy}}{\sqrt{S_{xx}S_{yy} - S_{xy}^2}}$

$S_{yy} = 5781 - 10(23.9)^2 = 68.9$

$\Rightarrow$ $t = \frac{\sqrt{8}\,(82)}{\sqrt{168(68.9) - 82^2}} = \frac{231.931}{69.65} = 3.33$

$\Rightarrow$ p-value = 0.0104 (using R)

Therefore, we reject $H_0$ since p-value is small.

c) $r = \hat{\beta}\sqrt{\frac{S_{xx}}{S_{xy}}} = 0.488\sqrt{\frac{168}{68.9}} = \boxed{0.76}$

The lin. relationship is relatively strong.

d) $\hat{y}_{16} = 18.043 + 0.488(16) = 25.85$

$\Rightarrow$ 95% confidence interval is

$25.85 \pm 2.306\,(s)\sqrt{\frac{1}{10} + \frac{(16-12)^2}{168}}$

where $s = \sqrt{\frac{S_{yy} - \hat{\beta}S_{xy}}{n-2}} = \sqrt{\frac{68.9 - 0.488(82)}{8}} = 1.9$

$\Rightarrow$ PI is $25.85 \pm 1.94 = \boxed{(23.92\ ,\ 27.79)}$

e) The interval will be narrowest at $x = \bar{x}$