Tutorial week 11 Solutions

(1) $H_0: \rho = 0.5$ vs. $H_1: \rho > 0.5$

$r = 0.73$, $n = 9$

Under H_0, $Z(R) \approx N\left(\tanh^{-1}(0.5), \frac{1}{9-3}\right)$

p-value $= P(R \geq 0.73) = P(Z(R) \geq Z(0.73))$

$$\approx P\left(Z \geq \frac{\tanh^{-1}(0.73) - \tanh^{-1}(0.5)}{\sqrt{1/6}}\right)$$

$$= P\left(Z \geq \frac{0.92873 - 0.54931}{0.40825}\right) = P(Z \geq 0.93)$$

$$= 1 - 0.8238 = \boxed{0.1762}$$

Conclusion: Data are consistent with H_0.

(2) Assumptions: $(X, Y) \sim N_2\left(\begin{pmatrix}\mu_1 \\ \mu_2\end{pmatrix}, \begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}\right)$,

where X = heat of green wood
Y = heat of air-dried wood

95% CI for $\tanh^{-1}(\rho)$ is

$$\tanh^{-1}(0.94) \pm 1.96\sqrt{1/10}$$

$$= 1.73805 \pm 0.619806 = (1.1182, 2.3579)$$

$\Rightarrow$ 95% CI for ρ is

$$(\tanh(1.118), \tanh(2.358)) = \boxed{(0.81, 0.98)}$$

(3)

a) $\hat{m}(x_i) = \dfrac{\sum_{j=1}^{n} K\left(\frac{x_j - x_i}{h}\right) y_j}{\sum_{j=1}^{n} K\left(\frac{x_j - x_i}{h}\right)}$,

where $K(x) = I_{[-0.5, 0.5]}(x)$.

That is, $\hat{m}(x_i)$ is the average of those y_j's, for which the corresponding x_j's are within $\pm(0.5)h$ distance of x_i.

b) $h = 0.01$

How many x_j's are within $(0.01)(0.5) = 0.005$ of $x_{26} = 0.568$, i.e. in the interval $[0.563, 0.573]$?

Answer: Only $y_{26} = 0.374$ is used.

c) $h = 0.1 \Rightarrow 0.5(h) = 0.05 \Rightarrow$ int. is $[0.518, 0.618]$

There are 7 points in it: $x_{87}, x_{86}, x_{26}, x_{85}, x_{14}, x_{72}, x_{84}$

e) $\hat{m}(x_{26}) = \dfrac{y_{87} + y_{86} + y_{26} + y_{85} + y_{14} + y_{72} + y_{84}}{7}$

$= \dfrac{0.53 + 0.37 + 0.374 + 0.678 + 1.192 + 0.423 + 0.563}{7}$

$= \dfrac{4.63}{7} = \boxed{0.661}$.

d) $0.3(0.5) = 0.15 \Rightarrow$ int. is $[0.418, 0.718]$

$\Rightarrow$ 18 points are used.

(4) $\sum_{j=1}^{n} (\hat{y}_j - \hat{y}_{j(i)})^2 = (\hat{y} - \hat{y}_{(i)})^T (\hat{y} - \hat{y}_{(i)}) = (X\hat{\beta} - X\hat{\beta}_{(i)})^T (X\hat{\beta} - X\hat{\beta}_{(i)})$

$= [X(\hat{\beta} - \hat{\beta}_{(i)})]^T [X(\hat{\beta} - \hat{\beta}_{(i)})] = (\hat{\beta} - \hat{\beta}_{(i)})^T X^T X (\hat{\beta} - \hat{\beta}_{(i)})$.