Tutorial week 12 solutions

(1) We first estimate λ by its MLE:

$$\hat{\lambda} = \bar{X} = \frac{24(0) + 16(1) + 16(2) + 18(3) + \ldots + 1(12)}{120} = \boxed{3.167}$$

Then the prob. under $Po(3.167)$ model are:

0	1	2	3	4	5	6	7	8	9	10	11	12
0.042	0.133	0.211	0.223	0.177	0.112	0.059	0.0267	0.0106	$3.7(10^{-3})$	$1.18(10^{-3})$	$3.39(10^{-4})$	$8.95(10^{-5})$

with expected counts:

too small

5.06	16.01	25.36	26.77	21.19	13.42	7.08	3.2	1.27	0.45	0.14	0.04	0.01

We need to combine the last 6 categories into one cell:

Group size: [7–12]

Obs. count: 16

Exp. count: 5.11

Then the χ^2 T.S. for testing H_0: Data are Poisson is

$$\chi^2 = \frac{(24-5.06)^2}{5.06} + \frac{(16-25.36)^2}{25.36} + \frac{(16-16.01)^2}{16.01} + \ldots + \frac{(16-5.11)^2}{5.11}$$

$= \boxed{103.86}$

$\Rightarrow$ p-value $= P(\chi^2_6 \geq 103.86) \approx 0$

Conclusion: Reject H_0 and conclude the Poisson model is not plausible.

(4) Problem 19, p. 536 from Rice

The contingency table with totals is

		Total
12	5	17
4	9	13
Total 16	14	

p-value $= P(X \geq 12)$ where $X \sim HG(17, 13, 16)$, i.e. $P(X=x) = \frac{\binom{17}{x}\binom{13}{16-x}}{\binom{17+13}{16}}$

In R: 1 − phyper(11, 17, 13, 16) = 0.035

= phyper(4, 13, 17, 16)

= 1 − phyper(8, 13, 17, 14)

= ...

Also try fisher.test

(2) Ho: $p_{1j} = p_{2j} = p_{3j}$, $j=1,2$ vs. H₁: The prob. are different

T.S. $$\chi^2 = \sum_i \sum_j \frac{\left(n_{ij} - \frac{n_{i\cdot} n_{\cdot j}}{n}\right)^2}{\frac{n_{i\cdot} n_{\cdot j}}{n}}$$

The expected counts are $\frac{n_{1\cdot} n_{\cdot 1}}{n} = \frac{32(62)}{103} = 19.26$, $\frac{n_{1\cdot} n_{\cdot 2}}{n} = 12.74$,

$\frac{33(62)}{103} = 19.86$, 13.14, 22.87 and 15.13

$$\Rightarrow \chi^2 = \frac{(14-19.26)^2}{19.26} + \frac{(18-12.74)^2}{12.74} + \dots + \frac{(15-15.13)^2}{15.13} = \boxed{6.95}$$

p-value $= P(\chi^2_2 \geq 6.95) = \boxed{0.031}$

Conclusion: IQ differs in the 3 income groups.

(3) Ho: Age and watching violence are ind. variables

H₁: They are dependent

T.S. and expected counts calculated as above:

$\frac{41(26)}{81} = 13.16$, $\frac{41(27)}{81} = 13.67$, 14.17, 12.84, 13.33, 13.83

$$\Rightarrow \chi^2 = \frac{(8-13.16)^2}{13.16} + \frac{(12-13.67)^2}{13.67} + \dots + \frac{(7-13.83)^2}{13.83} = \boxed{11.17}$$

$\Rightarrow$ p-value $= P(\chi^2_2 \geq 11.17) = \boxed{0.004}$

Conclusion: Very strong evidence against Ho