STAT2912: Statistical Tests

Semester 2	Statistical Tests (Advanced)	2011

Exercise 1 Solutions

1. Recall the following facts:

$$\mathbf{P}(Z \le x) = \Phi(x); \quad \Phi(-x) = 1 - \Phi(x), \text{ for } x \ge 0;$$
$$\mathbf{P}(y \le Z_1 \le x) = \Phi(x-1) - \Phi(y-1), \text{ for any } x > y;$$
$$\mathbf{P}(t_v \le -x) = P(t_v \ge x); \quad \mathbf{P}(|t_v| \le x) = 2\mathbf{P}(t_v \le x) - 1.$$

(a)

$$\mathbf{P}(Z \le 1.25) = 0.8943; \quad \mathbf{P}(0.5 < Z_1 \le 1.23) = \Phi(0.23) - \Phi(-0.5) = 0.2825;$$
$$\mathbf{P}(t_3 \le 2.353) = 1 - .05 = 0.95; \quad \mathbf{P}(|t_5| \le 1.476) = 1 - 2 \times 0.1 = 0.8.$$

(b) $1.86,\ 0.711,\ -1.65,\ 1.96$

2. (i) The hypothesis to be tested is

$$H_0: \ \mu = 800 \quad \text{vs} \quad H_1: \ \mu < 800.$$

(ii) There are only five measurements on which to base the test. Let $X_j, 1 \le j \le 5$ denote the random variables that generate the observed values $785, 805, 790, 793$, and 802. We have to assume that $X_1, X_2, ..., X_5$ are iid normal random variables.

(iii) A t-test must be employed. The test statistic is $T_n = \sqrt{5}(\bar{X} - 800)/S$. The calculation of the p-value is as follows:

$$\bar{x} = \frac{785 + 805 + 790 + 793 + 802}{5} = 795; \quad s^2 = (\sum_j x_j^2 - n\bar{x}^2)/(n-1) = 69.5$$
$$t = \sqrt{5}(\bar{x} - 800)/s = -1.341$$
$$p = \mathbf{P}(T_n \le t) = \mathbf{P}(t_4 \le -1.341) = 0.1255026.$$

(iv) The data are consistent with the null hypothesis, that is, there is no indication that the average yield is less than 800 tons.

3. (a) Let θ be the proportion of black jurors selected in a panel. If the selection is random, then the θ should be 0.26. Assume we are looking for evidence of discrimination. Test

$$H_0: \theta = 0.26 \quad \text{vs} \quad H_1: \theta < 0.26.$$

Let Y be the number of black jurors in the panel. Then $Y \sim B(100, \theta)$, n is large. We can use the test statistic:

$$T_n = \frac{Y - 100 \times 0.26}{\sqrt{100 \times 0.26 \times 0.74}}.$$

The observed value is

$$t_{\mathrm{obs}} = \frac{8 - 100 \times 0.26}{\sqrt{100 \times 0.26 \times 0.74}} = -4.104,$$

and hence the p-value

$$p \approx \mathrm{P}(Z < t_{\mathrm{obs}}) = \Phi(t_{\mathrm{obs}}) = 0.00002.$$

There is very strong evidence that the selection underrepresented the blacks.

4. The rejection region is $|t| > t_{15,0.025}$, where, from the t-table, $t_{15,0.025} = 2.131$

   The observed value of the test statistic is $t = \frac{9.41}{12.5/\sqrt{16}} = 3.011$.

   Conclusion: Since $|3.011| > 2.131$ we reject H_0 and conclude that there is enough evidence, at the 0.05 level, that the average spoilage time differs for the two preservatives.