

Exercise 6 Solutions

1. (a) We first calculate the different SS's:

$$\begin{aligned}
 SS_T &= 33180 - 32279.08 = 900.92 \\
 SS_B &= \frac{270^2}{9} + \frac{275^2}{10} + \frac{295^2}{10} + \frac{282^2}{10} - 32279.08 = 38.32 \\
 SS_W &= 900.32 - 38.32 = 862.6.
 \end{aligned}$$

Now we combine everything together in the ANOVA table:

	df	SS	MS	F	p-value
Groups	3	38.32	12.77	0.5183	0.6724
Residual	35	862.6	24.65		
Total	38	900.92			

Since the p-value > 0.1 , we fail to reject H_0 . That is, there is insufficient evidence that the average heartbeat is different for the 4 age groups.

- (b) The test statistic is

$$t = \frac{30 - 27.5}{\sqrt{24.65 \left(\frac{1}{9} + \frac{1}{10}\right)}} = 1.1.$$

Based on 35 df, we find the p-value is

$$p = 2P(T_{35} > 1.1) = 0.28.$$

Conclusion: Since $p > 0.1$ we conclude there is no significant difference between the age groups 10-19 and 20-39.

2. (a) Preliminary calculations:

$$\begin{aligned}
 T_1 &= 30.4 \\
 T_2 &= 23.9 \\
 T_3 &= 29.2 \\
 CM &= \frac{83.5^2}{15} = 464.82 \\
 SS_T &= 5.43 \\
 SS_B &= 4.78 \\
 SS_W &= 0.65.
 \end{aligned}$$

Now we combine everything together in the ANOVA table:

	df	SS	MS	F	<i>p</i> -value
Groups	2	4.78	2.39	44.31	2.9×10^{-6}
Residual	12	0.65	0.054		
Total	14	5.43			

Conclusion: Since $p\text{-value} < 0.01$ we have very strong evidence against H_0 . That is, there is sufficient evidence to indicate that the mean dissolved content is different for the 3 locations.

(b) The test statistic is

$$t = \frac{0.24}{\sqrt{0.054 \left(\frac{2}{5}\right)}} = 1.63.$$

Based on 12 df, the p -value is $p = 2P(T_{12} > 1.63) = 0.13$. There is insufficient evidence that mean dissolved content is different between Location 1 and Location 3.

(c) We already have $t_{13} = 1.63$ from part (b). The other two test statistics are:

$$t_{12} = \frac{6.08 - 4.78}{\sqrt{0.054 \left(\frac{2}{5}\right)}} = 8.85$$

and

$$t_{23} = \frac{4.78 - 5.84}{\sqrt{0.054 \left(\frac{2}{5}\right)}} = -7.21.$$

The unadjusted p -values are $p_{13} = 0.13$, $p_{12} = 2P(T_{12} > 8.85) = 1.33 \times 10^{-6}$ and $p_{23} = 2P(T_{23} > |-7.21|) = 1.07 \times 10^{-5}$. The Bonferroni adjustment requires that we multiply each p -value by $\frac{3(3-1)}{2} = 3$. That is, the adjusted p -values are $p_{13} = 0.39$, $p_{12} = 3.99 \times 10^{-6}$ and $p_{23} = 3.21 \times 10^{-5}$.

Conclusion: Location 2 is different than each of Locations 1 and 3, but Locations 1 and 3 are not significantly different from each other in terms of average dissolved oxygen.