Semester 2	Statistical Tests (Advanced)	2011

Exercise 7 Solutions

1. Notice that the data are not ranks, so we must first rank them.

   Ranked combined sample:

Ordered data	1	2	3	4	5	7	7	8	9	11	12	13	14	15	16
Ranks	1	2	3	4	5	6.5	6.5	8	9	10	11	12	13	14	15
Treatment	2	1	2	1	3	1	2	2	3	1	3	2	3	1	3

   Now we have:

$$\begin{aligned} R_{1\cdot} &= 2 + 4 + 6.5 + 10 + 14 = 36.5 \\ R_{2\cdot} &= 30.5 \\ R_{3\cdot} &= 53 \\ \sum_{i=1}^{3} \frac{R_{i\cdot}^2}{n_i} &= \frac{36.5^2}{5} + \frac{30.5^2}{5} + \frac{53^2}{5} = 1014.3 \\ N &= 15, \end{aligned}$$

   and the test statistic is

$$\begin{aligned} K^* &= \frac{\frac{12}{N(N+1)}\left(\sum_{i=1}^{g} \frac{R_{i\cdot}^2}{n_i}\right) - 3(N+1)}{1 - \frac{\sum(d_i^3 - d_i)}{N^3 - N}} \\ &= \frac{\frac{12}{15(16)}(1014.3) - 3(15)}{1 - \frac{2^3-2}{15^3-15}} = \frac{2.715}{0.9982143} = 2.72. \end{aligned}$$

   The p-value is p-value $= \mathrm{P}(\chi_2^2 > 2.72) = 0.26$.

   Conclusion: Groups are not significantly different.

2. (a) The model is

$$X_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}, \quad , i = 1, 2, 3, \quad j = 1, 2,$$

where μ is an overall typical value, α_i is the "row effect", β_j is the "column effect", and ϵ_{ij} represents the departure of y_{ij} from the additive model.
We want to find $m, a_i, i = 1, \ldots, 3, b_j, j = 1, 2$ that minimise

$$\text{SAR} = \sum_i \sum_j |y_{ij} - m - a_i - b_j|,$$

where SAR stands for "sum of absolute residuals". There is no formula for the exact solution, but can obtain an approximation by the median polish iterative procedure.

(b) Here is the sequence of steps:

3.7	5.9	
8.2	9.6	
11.7	12.6	
8.2	9.6	8.9

↓

-4.5	-3.7	-4.1
0	0	0
3.5	3	3.25
-0.7	0.7	8.9

↓

-0.4	0.4	-4.1
0	0	0
0.25	-0.25	3.25
-0.7	0.7	8.9

Estimates:

$$m : \hat{\mu} = 8.9$$
$$a : \hat{\alpha}_1 = -4.1, \hat{\alpha}_2 = 0, \hat{\alpha}_3 = 3.25$$
$$b : \hat{\beta}_1 = -0.7, \hat{\beta}_2 = 0.7.$$