

THE UNIVERSITY OF SYDNEY
MATH1003 INTEGRAL CALCULUS AND MODELLING

Semester 2

Practice Questions for Quiz 1

2009

Quiz 1 will be held during your tutorial in week 7 (beginning 7th September).

The quiz questions will be based on material covered during lectures in weeks 1–5, which corresponds to material covered in tutorials in weeks 2–6.

The quiz will run for 40 minutes. You may use a non-programmable calculator. No other materials are permitted.

1. Find upper and lower Riemann sums for $f(x) = 2x^2 - 1$ on $[1, 6]$, using 5 equal subintervals.

2. Find the following:

a) $\int x\sqrt{x} + x^3 dx$; b) $\int_0^{\frac{\pi}{2}} \cos x \sin 2x dx$;
c) $\int_0^1 xe^{6x} dx$; d) $\int x\sqrt{x^2 + 1} dx$;
e) $\int_0^2 x\sqrt{x+1} dx$.

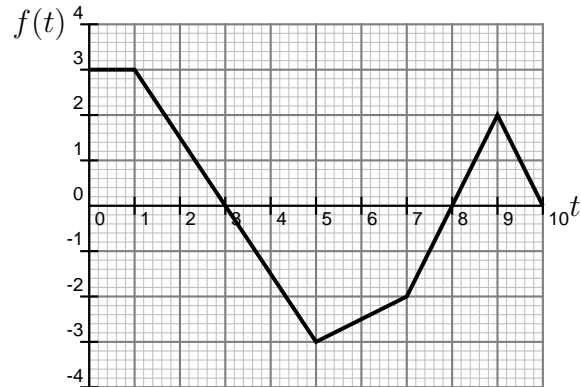
3. a) Evaluate $\int_0^{16} x^{3/2} dx$.

- b) Use upper and lower Riemann sums for $f(x) = x^{3/2}$ on $[0, 16]$ with 16 equal subintervals to find upper and lower bounds for $\sum_{n=1}^{16} n^{3/2}$.

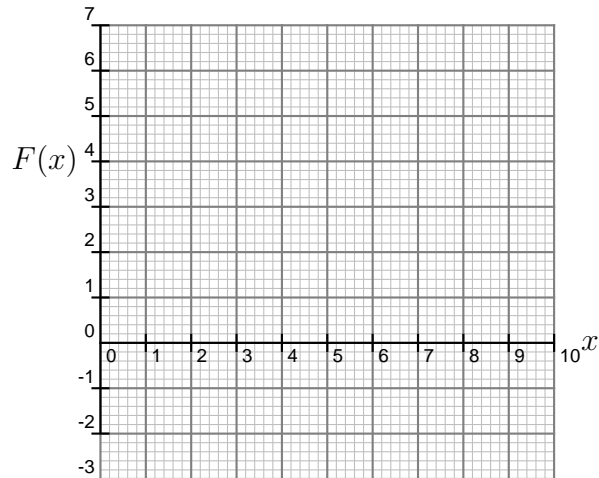
4. Given that $\int_0^2 3f(x) dx = 6$ and $\int_6^0 f(x) dx = 7$, determine $\int_2^6 3f(x) dx$.

5. a) Find the area enclosed by the curves $y = \sqrt{x}$ and $y = x^4$.
b) Find the volume of the solid formed when the area in part a) is rotated about
(i) the x -axis;
(ii) the y -axis;
(iii) the line $x = 2$.

6. Let $F(x) = \int_0^x f(t) dt$, for $0 \leq x \leq 10$, where f is the function whose graph is shown below.



- What is $F'(5)$?
- What is $F(7)$?
- For what values of x is the function $F(x)$ increasing?
- Determine the positions of the local minima and local maxima of the function $F(x)$. Give your answer in the form (x, y) .
- Plot the function $F(x)$ on the graph below.



7. a) Find a reduction formula for $I_n = \int x^n e^{2x} dx$.
- b) Hence calculate I_3 .

ANSWERS

1. 105, 175.

2. a) $\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^4 + C$;

b) $\frac{2}{3}$;

c) $\frac{5e^6 + 1}{36}$;

d) $\frac{(x^2 + 1)^{\frac{3}{2}}}{3} + C$;

e) $\frac{4(6\sqrt{3} + 1)}{15}$.

3. a) $\frac{2048}{5}$;

b) Upper estimate: $\frac{2368}{5}$, Lower estimate: $\frac{2048}{5}$.

4. -27.

5. a) $\frac{7}{15}$;

b) i) $\frac{7\pi}{18}$;

ii) $\frac{7\pi}{15}$;

iii) $\frac{7\pi}{5}$.

6. a) -3;

b) -2;

c) $0 \leq x \leq 3$ and $8 \leq x \leq 10$;

d) local minimum: $(8, -3)$, local maximum $(3, 6)$.

7. a) $I_n = \frac{x^n}{2}e^{2x} - \frac{n}{2}I_{n-1}$;

b) $I_3 = \frac{(4x^3 - 6x^2 + 6x - 3)e^{2x}}{8} + C$.