The University of Sydney
Math1003 Integral Calculus and Modelling

Semester 2 **Exercises for Assignment 1** 2009

This assignment is due by 4pm on Thursday 27 August 2009. It is worth 5% of the assessment for this unit of study. The assignment should be posted in the glass-fronted collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue. (NOT the glass-fronted collection boxes near the pyramids on Carslaw Level 3, nor the open wooden pigeonholes.) Please do not post your assignment before 27 August, since the boxes are also used for the collection of assignments in other units. Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. A cover sheet must be signed and attached.

1. (i) [3 Marks] Sketch the curve $y = 9-x^2$ over the interval $[0,3]$. Give the expression of the area under the curve in terms of a definite integral. Then use the Fundamental Theorem of Calculus to calculate the area under the curve.

(ii) [2 Marks] Evaluate the lower Riemann sum for $f(x) = 9 - x^2$ on $[0,3]$ by dividing this interval into 4 subintervals of equal length.

(iii) [2 Marks] Divide the interval $[0,3]$ into n subintervals of equal length. Prove that for this partition of $[0,3]$ the lower Riemann sum L_n for $f(x) = 9 - x^2$ equals

$$L_n = \frac{3}{n}\sum_{k=1}^{n}\left[9 - \left(\frac{3k}{n}\right)^2\right].$$

(iv) [1 Mark] Without evaluating the limit directly, explain why as n tends to ∞, the limit of the lower Riemann sum L_n equals 18. (Hint: Use a theorem from lectures.)

2. Let f be a continuous function on $[-a,a]$ which is an odd function, that is

$$f(-x) = -f(x) \quad \text{for all } x \text{ in the interval } [-a,a].$$

(a) [1 Mark] Give a geometric interpretation of why $\int_{-a}^{a} f(x)\,dx = 0$. (Give a sketch.)

(b) [2 Marks] By using a suitable substitution, prove that

$$\int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx,$$

whenever f is a continuous and odd function on $[-a,a]$.

(c) [1 Mark] Evaluate the integral $\int_{-\pi/2}^{\pi/2} (\sin x)^5\,dx$.

3. (i) [1 Mark] Sketch the area enclosed by the positive y axis, the line $y = 1$ and the curve $y = x^3$.

(ii) A solid of rotation S is formed by rotating this area about the y-axis.

(a) [1 Mark] For each y in $[0,1]$ on the y-axis, the cross sectional area of S perpendicular to the y-axis is denoted by $A(y)$. Find a formula for $A(y)$.

(b) [2 Marks] Derive an expression approximating the volume ΔV of the thin solid obtained by rotating a thin horizontal strip of the area of width Δy located at y.

(c) [4 Marks] Hence determine a definite integral which will give the total volume of the solid and evaluate this integral.