

THE UNIVERSITY OF SYDNEY  
MATH1003 Integral Calculus and Modelling

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Semester 2

Practice Assignment 2

2009

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This assignment is NOT to be handed in. It is a selection of questions for you to use as a study guide. Detailed solutions are provided so that you can compare your solutions with model solutions. You will gain a greater benefit if you make a serious attempt at the problems before looking at the solutions.

1. (a) [3 marks] Find the general solution to  $(x^2 + x - 2)\frac{dy}{dx} = xy^2$ .  
(b) [3 marks] Find the particular solution to  $\frac{dy}{dx} - y \tan x = x$ , given that  $y(0) = 0$ .
  
2. A simple physiological model suggests that an adult male athlete needs 40 calories per day per kilogram of body weight in order to maintain his weight. If he consumes more, or fewer, than this amount of calories, then his weight changes at a rate which is proportional to the difference between the number of calories consumed, and the number required to maintain his weight. The constant of proportionality is  $\frac{1}{7000}$  (kilograms per calorie). Let “P” denote a particular male athlete who has a constant intake of  $L$  calories per day.
  - (a) [2 marks] Assuming this model, write a differential equation that describes the rate of change of P’s weight. (Don’t forget to name your variables.)
  - (b) [3 marks] Solve your differential equation.
  - (c) [2 marks] If P currently weighs 80 kilograms, and consumes 3000 calories a day for the next 90 days, what will his weight be at the end of those 90 days?
  
3. A tank with a capacity of 1000 litres contains 600 litres of a mixture of water and chlorine. The concentration of the chlorine is 0.05 grams per litre. In order to reduce the chlorine concentration, fresh water is pumped into the tank at a rate of 6 litres per second for 3 minutes. The mixture in the tank is kept well-stirred, and is pumped out at the rate of 4 litres per second during the same 3 minutes.
  - (a) [4 marks] Find the mass of chlorine in the tank as a function of time.
  - (b) [3 marks] What is the concentration of chlorine in the tank at the end of the 3 minutes?