MATH1003 2007 Exam, Selected Answers

1. (*a*) *(i)* $\dfrac{4}{3}\dfrac{1}{x-1}+\dfrac{1}{3}\dfrac{2x-1}{x^2+2}$.

(ii) $\dfrac{1}{3}\left\{\ln(x-1)^4(x+2)^2-\dfrac{1}{\sqrt{2}}\tan^{-1}\dfrac{x}{\sqrt{2}}\right\}+C$.

(*b*) $\dfrac{1}{9}$.

(*c*) $\dfrac{1}{3}\sinh^{-1}\dfrac{3x}{4}+C$.

2. (*a*) First order linear. Not separable.

General Solution $y=\dfrac{1}{x}\ln|x|+7+\dfrac{C}{x}$.

Particular Solution $y=\dfrac{1}{x}\ln x+7-\dfrac{7}{x}$.

(*b*) Separable.

General Solution $y=\ln(x-\cos x+C)$.

Particular Solution $y=\ln(x-\cos x+2)$.

(*c*) General Solution $y=e^{2x}(C\cos x+D\sin x)$.

Particular Solution $y=3e^{2x}\sin x$.

3. (*a*) $\ln 2$.

(*b*) Area is infinite.

(*c*) Volume of revolution $V=\dfrac{9\pi}{4}-2\sqrt{2}\pi$.

4. (*a*) *(i)* 500 g/hr.

(ii) $\dfrac{x}{100}$ g/hr.

(iii)

(*b*) *(i)* $x=50000-Ae^{-t/100}$.

(ii) $x=50000-40000e^{-t/100}$.

(*c*) *(i)* $1000(5-4e^{-1/5})=17251$ g (to 5 sig. figures).

(ii) $10(5-4e^{-12})\approx 50$ g/m^3.

5. (*a*) $L_n=\displaystyle\sum_{k=1}^{n}\frac{1}{k+1}=\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n+1}$.

$U_n=\displaystyle\sum_{k=1}^{n}\frac{1}{k}=1+\frac{1}{2}+\cdots+\frac{1}{n}$.

$\displaystyle\lim_{n\to\infty}(U_n-L_n)=1$.

(*b*) *(i)* $g'(x)=-\dfrac{1}{x^2}\left((1-\sin(2\pi x)+\dfrac{2\sin^2(\pi x)}{\pi x}\right)$.

(ii) Bound $C=\frac{1}{\pi}$, follows from integrating $g(t)\le\dfrac{1}{t}+\dfrac{1}{\pi t^2}$.