The University of Sydney
School of Mathematics and Statistics

Week 1 Exercises and Objectives

MATH1003: Integral Calculus and Modelling — Semester 2, 2015

Web Page: www.sydney.edu.au/science/maths/u/UG/JM/MATH1003/
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Assumed Knowledge: Sigma notation for sums. The ideas of a sequence of numbers and of the limit of a sequence. Sketching a curve given a function $f(x)$ or a set of tabulated values of a function $f(x)$.

Objectives:

(1a) To know what is meant by a Riemann sum.

(1b) To understand the definition of the definite integral of a function as the limit of a Riemann sum.

(1c) To understand how to interpret a definite integral of a function in terms of area when the function is not always positive.

(1d) To be able to use the upper and lower sums to obtain an estimate of the error in evaluating a definite integral.

(1e) To be able to find the number of subintervals (sampling frequency) required to reduce the error to a given value in the case of monotonic functions.

Exercises:

Note: The first three questions are practice in skills that are needed for this unit, that should already have been learned in school. The remaining three questions require knowledge from the Week 1 lecture material.

1. Simplify

(a) $e^{\ln x - 2\ln y}$ (b) $\ln\left(\dfrac{e^x}{e^y}\right)$

2. (a) Rearrange the equation $\dfrac{1}{y} - y = \dfrac{1}{x}$ to find x as a function of y.

(b) Rearrange the equation $\sqrt{\dfrac{1}{y} - \dfrac{1}{3}} = \dfrac{2}{t}$ to find y as a function of t.

(c) Rearrange each of the following to find t:

(i) $N = N_0 e^{kt}$ (ii) $x = 4 + 2\ln t$

3. Find the derivatives of the following functions:

(a) $(x+3)^2$

(b) $x \ln x$

(c) $(x^3+1)^{10}$

(d) $\cos(x^2)$

(e) $e^{\cos x}$

(f) $\dfrac{\cos x}{2+\sin x}$

(g) $\dfrac{1}{\sqrt{4-x^2}}$

(h) $x\sqrt{x^2-4}$

4. Evaluate the sum $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k+2}$.

5. If $c(t)$ represents the cost, in dollars per day, to heat your house, where t is measured in days and $t=0$ on 1 June 2005, what does $\int_0^{90} c(t)\,dt$ represent?

6. Draw diagrams illustrating the approximation of $\ln 2 = \int_1^2 \frac{dt}{t}$ using upper and lower Riemann sums with 10 equal subdivisions.
Estimate $\ln 2$ correct to one decimal place by calculating the upper and lower Riemann sums.