Assumed Knowledge: Finding the roots of quadratic equations. Euler’s formula $e^{i\theta} = \cos \theta + i \sin \theta$.

Objectives:

(11a) To be able to write down the auxiliary (or characteristic) equation associated with a second-order differential equation with constant coefficients.

(11b) To be able to construct the solutions to such differential equations in terms of exponential and trigonometric functions.

Exercises:

1. Find the general solution of $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 4y = 0$.

2. Find the particular solution of $\frac{d^2y}{dt^2} - 9y = 0$ which satisfies the initial conditions $y = 3$ and $\frac{dy}{dt} = 3$ when $t = 0$.

3. Find the general solutions of these second-order homogeneous equations. In each case give your answer in terms of real functions.
   
   (a) $2\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 3y = 0$.
   
   (b) $\frac{d^2y}{dx^2} + 3y = 0$.
   
   (c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

4. Find the general solution of each of these second-order linear homogeneous equations. Hence find the particular solution for the given conditions. In each case give your answer in terms of real functions.
   
   (a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 20y = 0$, $y(0) = y'(0) = 1$.
   
   (b) $\frac{d^2y}{dx^2} + 9y = 0$, $y(0) = 1$, $y(\pi/6) = 3$.
   
   (c) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = 0$, $y(0) = 0$, $\frac{dy}{dt} = 3$ when $t = 0$.
   
   (d) $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 0$, $x(0) = 1$, $x(1) = 3e^2$. 