The University of Sydney
School of Mathematics and Statistics

Week 11 Exercises and Objectives

MATH1003: Integral Calculus and Modelling — Semester 2, 2015

Web Page: www.sydney.edu.au/science/maths/u/UG/JM/MATH1003/
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Assumed Knowledge: Finding the roots of quadratic equations. Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$.

Objectives:

(11a) To be able to write down the auxiliary (or characteristic) equation associated with a second-order differential equation with constant coefficients.

(11b) To be able to construct the solutions to such differential equations in terms of exponential and trigonometric functions.

Exercises:

1. Find the general solution of $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 4y = 0$.

2. Find the particular solution of $\dfrac{d^2y}{dt^2} - 9y = 0$ which satisfies the initial conditions $y = 3$ and $\dfrac{dy}{dt} = 3$ when $t = 0$.

3. Find the general solutions of these second-order homogeneous equations. In each case give your answer in terms of real functions.

(a) $2\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 3y = 0$.

(b) $\dfrac{d^2y}{dx^2} + 3y = 0$.

(c) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$.

4. Find the general solution of each of these second-order linear homogeneous equations. Hence find the particular solution for the given conditions. In each case give your answer in terms of real functions.

(a) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 20y = 0, \quad y(0) = y'(0) = 1$.

(b) $\dfrac{d^2y}{dx^2} + 9y = 0, \quad y(0) = 1, \quad y(\pi/6) = 3$.

(c) $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 7y = 0, \quad y(0) = 0, \quad \dfrac{dy}{dt} = 3$ when $t = 0$.

(d) $\dfrac{d^2x}{dt^2} - 4\dfrac{dx}{dt} + 4x = 0, \quad x(0) = 1, \quad x(1) = 3e^2$.