

Tutorial 1 Week 2

1. Which of the following strings of brackets are balanced? In each case, explain carefully why the string is, or is not, balanced:
 - (i) $((())$
 - (ii) $(())()$
 - (iii) $()()$
2. Suppose you have numbers $x_0, x_1, x_2, \dots, x_n$ and you want to multiply them together. In how many ways can you insert brackets into the string $x_0x_1x_2\dots x_n$ so that the order of multiplication is completely specified? Each pair of brackets should contain just two terms. For example, when $n = 2$, there are two ways: $(x_0(x_1x_2))$ and $((x_0x_1)x_2)$. Try $n = 3$ and $n = 4$.
3. Consider a river system with n sources which eventually merge to form a single stream. Assuming that no more than two streams merge at any point, we are interested in the number of ways that the mergers can take place.
 - (i) Compile a table of values for $n = 1, 2, 3$, and 4 and then find (guess) a general formula.
 - (ii) If possible, find a connection with the bracketing problem in Question 2.
4. In how many ways can a convex polygon with $n + 1$ sides (labelled $0, 1, 2, \dots, n$) be divided into triangles by non-intersecting diagonals? If possible, find a connection with the bracketing problem in Question 2.
5. For $n \geq 0$, evenly distribute $2n$ points on the circumference of a circle. Let a_n be the number of ways in which these $2n$ points can be paired off as n chords where no two chords intersect.
 - (i) Find a_n for $n = 2, 3$ and then find (or guess) a general formula.
 - (ii) If possible, find a connection with the first bracket problem [Problem 1.1].
6. For each of the following balanced strings of brackets,
 - (i) LLRLRLLRRLRLR (ii) LLRLRLRRLRLLRLLRRLRRLR;
 - (a) draw the corresponding planar diagram;
 - (b) construct the corresponding way of pairing off $2n$ points as n chords where no two chords intersect.

Problem Set 1

1. (i) Which of the following strings of brackets are balanced? In each case, explain carefully why the string is, or is not, balanced:

(a) $((()())$ (b) $((())(())$ (c) $((()))($

- (ii) Construct a balanced string of brackets corresponding to the following planar diagram or smiling face:



- (iii) Construct the planar diagram or smiling face corresponding to the following balanced string of brackets $((()())(())(())$.

2. Given two rows of boxes with n boxes in each row:

		\dots	
		\dots	

In how many ways can you place the numbers $1, 2, \dots, 2n$ in the boxes so that the numbers increase from left to right and so that each number in the bottom row is larger than the number in the box above it? Write down all the arrangements for $n = 1, 2, 3$ and 4 . Any conjectures?

3. For the following balanced string of brackets

LLRLRLLRRLRLR,

- (i) construct the corresponding planar diagram or smiling face;
- (ii) construct the corresponding way of pairing off 14 points on the circumference of a circle, as 7 chords where no two chords intersect. [i.e., the hand-shaking problem for 14 people]
- (iii) construct the corresponding $(7, 7)$ tableau; that is, given 2 rows of boxes with 7 boxes in each row, place the numbers $1, 2, \dots, 14$ in the boxes so that the numbers increase from left to right and so that each number in the bottom row is larger than the number in the box above it.
4. In a soccer match between Teams A and B, the final score is n -all. At no time during the match was Team B in the lead. In how many different ways can the A-B scores be built up, starting at 0-0 and ending at n - n ? Try the cases $n = 1, 2, 3$ and write down the scoring patterns. Can you guess what the answer might be when $n = 4$? Find a connection between the scoring patterns and Problem 1.1.