

Tutorial 2 Week 3

1. Use the notation of set theory to describe:

- (i) The set of all odd integers between 2 and 10.
- (ii) The set of all odd integers between 2 and 200.
- (iii) The set of all odd integers.
- (iv) The set of integers divisible by 4.

For this exercise, do *not* use the dots notation (like ...).

2. Which of the following statements are true?

- (i) $\{2, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}$.
- (ii) $\{2\} \subseteq \{1, 2, 3, 4, 5, 6\}$.
- (iii) $2 \subseteq \{1, 2, 3, 4, 5, 6\}$.
- (iv) $2 \in \{1, 2, 3, 4, 5, 6\}$.
- (v) $\{2\} \in \{1, 2, 3, 4, 5, 6\}$.

Give reasons for your answers.

3. Let $A = \{1, 2, 3, \{2\}, \{2, 3\}, 4\}$. Which of the following statements are true?

- (i) $\{2\} \in A$.
- (ii) $\{\{2\}\} \subseteq A$.
- (iii) $\{2, \{2\}\} \subseteq A$.
- (iv) $\{2, \{3\}\} \subseteq A$.
- (v) $\{2, 3\} \in A$.
- (vi) $\{3, \{2, 3\}\} \subseteq A$.

4. Write out the following sets, where $A = \{a, b, c, \{a, d\}\}$:

- (i) $A \cup \{b, d, e\}$.
- (ii) $A \cap \{b, d, e\}$.
- (iii) $A \setminus \{a, b\}$.
- (iv) $A \setminus \{c, d\}$.
- (v) $A \setminus \{\{a, d\}\}$.
- (vi) $A \setminus \{a, \{a, d\}\}$.

Then write down the sizes of each of the sets.

5. List the elements in each of the six sets P , Q , $P \cup Q$, $P \cap Q$, $P \setminus Q$ and $Q \setminus P$, where

$$P = \{x \mid x \in \mathbb{Z} \text{ and } 4 \leq x \leq 10\},$$

$$Q = \{y \mid y \in \mathbb{Z} \text{ and } \frac{y}{2} \in \mathbb{Z} \text{ and } 0 \leq y^2 \leq 50\}.$$

6. Let $A = \{a, b, c, d\}$. Write down all the subsets of A . How many are there?

7. If A and B are subsets of a set X , prove that

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B).$$

Problem Set 2

1. Let $A = \{a, b, c, \{b\}\}$. Determine which of the following statements are true?
(i) $\{b\} \in A$; (ii) $\{b\} \subseteq A$; (iii) $\{\{b\}\} \subseteq A$; (iv) $\{a\} \in A$.
2. Let $A = \{x \mid x \in \mathbb{Z}, -3 < x \leq 2\}$ and $B = \{x^2 + 1 \mid x \in \mathbb{Z}, -3 < x \leq 2\}$.
(i) Write down the elements of A and B .
(ii) Find $A \cup B$, $A \cap B$ and $A \setminus B$. (iii) Find $|A \cup B|$, $|A \cap B|$ and $|A \setminus B|$.
3. Let $A = \{x \in \mathbb{N} \mid 1 \leq x^2 \leq 30\}$
and $B = \{x \in \mathbb{Z} \mid x = 2y \text{ for some } y \in \mathbb{Z} \text{ and } x^2 < 50\}$.
(i) List the elements of A and B . (ii) Find $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$.
(iii) Find $|A \cup B|$, $|A \cap B|$, $|A \setminus B|$, $|B \setminus A|$.
4. (i) Write the following using set notation:
The set G is the set of all odd integers which are greater than 22 and are not divisible by 5.
(ii) With G as described in part (i), classify each of the following statements as true or false, giving reasons.
(a) $G \subseteq \mathbb{Z}$, (b) $\mathbb{Z} \subseteq G$, (c) $G \cap \mathbb{Z} \neq \emptyset$
(d) $\mathbb{Z} \setminus G$ is the set of all even integers less than 22 which are divisible by 5.
5. Let $A = \{a, b, c\}$, $B = \{a, \{a\}, \{b, c\}\}$ and $C = \{\{a, b\}, b, c\}$.
(i) What are $|A|$, $|B|$ and $|C|$?
(ii) Write down $A \cup B$, $A \cup B \cup C$, $A \cap B$, $A \cap C$ and $B \cap C$.
(iii) Write down $A \setminus B$, $B \setminus A$, $A \setminus C$, $C \setminus A$, $B \setminus C$ and $C \setminus B$.
(iv) Which of the following statements are true? Give reasons!
(a) $A \subseteq B$ (b) $B = C$
(c) $\{a\} \in A$ (d) $\{a\} \in B$
(e) $\{\{a\}\} \subseteq B$ (f) $A \subseteq C$
(g) $\{a, b\} \subseteq A$ (h) $\{a, b\} \subseteq C$
(i) $\{a, b\} \in C$ (j) $\{\{a, b\}\} \subseteq C$
6. Let A , B and C be any three sets. Prove that
$$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C).$$
7. Prove that if A and B are subsets of X , then
$$X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B).$$