THE UNIVERSITY OF SYDNEY

MATH 1004

Second Semester	Discrete Mathematics	2011

Tutorial 9 Week 10

1. Draw truth tables for the following propositions.

(*i*) $\sim(p \vee q)$.

(*ii*) $\sim p \wedge \sim q$.

(*iii*) $p \wedge (p \Rightarrow q)$.

2. Show that the following pairs of propositions are equivalent:

(*i*) $(p \vee q) \Rightarrow r$; $(\sim p \wedge \sim q) \vee r$.

(*ii*) $p \Rightarrow \sim q$; $q \Rightarrow \sim p$.

3. Decide whether the following propositions are true or false:

(*i*) If $1 + 1 = 2$, then $2 + 2 = 5$.

(*ii*) If $1 + 1 \neq 2$, then pigs might fly.

(*iii*) If $1 + 1 = 2$, then $2 + 3 = 5$.

4. Show that $\sim(p \vee q) \vee (\sim p \wedge q) \vee p$ is a tautology.

5. Show that $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction.

6. Taking the universal set to be the set $\mathbb{R}$ of all real numbers, determine the truth or falsity of the following sentences.

(*i*) $(\forall x)\big((x \in \mathbb{Z}) \Rightarrow (x^2 - x - 1 > 0)\big)$.

(*ii*) $(\exists x)\big((x \in \mathbb{Z}) \wedge (x^2 - x - 1 > 0)\big)$.

(*iii*) $(\forall x)\big((x^2 = 1) \Rightarrow (x = 1)\big)$.

(*iv*) $(\exists x)\big((x^2 = 1) \wedge (x = 1)\big)$.

7. Write the following propositions in symbolic form:

(*i*) The square of every real number is non negative.

(*ii*) There is an x in the set A which is not in the set B.

8. Write the following propositions in symbolic form:

(*i*) All hungry crocodiles are not amiable.

(*ii*) Some crocodiles, if not hungry, are amiable.

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Problem Set 9

1. *(i)* Show that $((p \vee q) \wedge \sim q) \Rightarrow p$ is a tautology.

(ii) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

2. For each of the following propositions, write down its truth table and determine whether or not it is a tautology or contradiction.

(i) $(p \wedge \sim q) \Rightarrow \ \sim(p \Rightarrow q)$

(ii) $(p \wedge \sim q) \wedge (\sim p \vee q)$

3. *(i)* Draw truth table for the proposition

$$(p \wedge q) \vee \sim(p \Rightarrow q),$$

and determine whether it is a tautology or a contradiction or neither.

(ii) Taking the universal set to be the set $\mathbb{R}$ of all real numbers, determine the truth and falsity of the following propositions.

(a) $(\forall x)((x > 2) \Rightarrow (x^2 > 4))$

(b) $(\forall x)((x^2 > 4) \Rightarrow (x > 2))$

(c) $(\exists x)((x > 2) \Rightarrow (x^2 > 4))$

(d) $(\exists x)((x^2 > 4) \Rightarrow (x > 2))$

4. *(i)* Draw a truth table for each of the following propositions, and determine whether it is a tautology or a contradiction or neither.

(a) $(p \Rightarrow q) \Rightarrow (\sim p \vee q)$,

(b) $(p \wedge q) \vee \sim(p \Rightarrow q)$.

(ii) Let $\mathbb{Z}$ be the set of all integers, $\mathbb{N}$ the set of all natural numbers and $\mathbb{R}$ the set of all real numbers. Determine the truth and falsity of the following propositions.

(a) $(\forall x \in \mathbb{Z})((x^2 = 9) \Rightarrow (x = 3))$

(b) $(\forall x \in \mathbb{N})((x^2 = 9) \Rightarrow (x = 3))$

(c) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})((x > 0) \Rightarrow (x = y^2))$

(d) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{R})((x > 0) \Rightarrow (x = y^2))$