THE UNIVERSITY OF SYDNEY

MATH 1004

Second Semester	Discrete Mathematics	2011

Tutorial 10 Week 11

Use induction to prove the following propositions.

For each of the given propositions, we let $S(n)$ be the given proposition, which is to be proved true for all integers greater than or equal to some specified integer, n_0 say. Then we show that

(a) $S(n_0)$ is true.

(b) $(\forall n \geq n_0)\ \big(S(n) \Rightarrow S(n+1)\big)$ is true. [To show (b), we suppose that $S(n)$ is true and, assuming that $n \geq n_0$, we prove that $S(n+1)$ is true.]

Then we conclude that $S(n)$ is true for all positive integers $n \geq n_0$.

1. Prove that $2^n \geq n + 12$, for all integers $n \geq 4$.

2. Prove that $1 + 3 + 5 + \cdots + (2n-1) = n^2$, for all positive integers n.

3. Prove that the sum of the first n positive even integers is $n^2 + n$.

4. Prove that $2 + 5 + 8 + \cdots + (3n-1) = \dfrac{n(3n+1)}{2}$, for all positive integers n.

5. Prove that 6 divides $n(n^2+5)$ for all positive integers n.

6. Prove that $11^n - 4^n$ is divisible by 7 for all positive integers n.

7. Prove that $5^n - 4n - 1$ is divisible by 16 for all positive integers n

8. Prove that for any integer $n \geq 1$, $\dfrac{(2n)!}{2^n}$ is an integer.

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Problem Set 10

1. Use Mathematical Induction to show that:

(*i*) The sum of the cubes of any three consecutive positive integers is divisible by 9.

(*ii*) For all positive integers n,

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}\,.$$

2. Use Mathematical Induction to show that:

(*i*) $n! > n^2$ for all integers $n \geq 4$.

(*ii*) $2^{2n+1}+1$ is divisible by 3 for all positive integers n.

3. Use Mathematical Induction to show that:

(*i*) For all positive integers n, $n! \geq 2^{n-1}$.

(*ii*) For all positive integers n,

$$\frac{(n+1)(n+2)\cdots(2n)}{2^n}$$

is an integer.

4. Use Mathematical Induction to show that:

(*i*) For any integers $n \geq 1$, $13^n - 5^n$ is divisible by 8 .

(*ii*) For any integer $n \geq 1$,

$$\frac{5}{1\cdot 2\cdot 3}+\frac{6}{2\cdot 3\cdot 4}+\cdots+\frac{n+4}{n(n+1)(n+2)}=\frac{n(3n+7)}{2(n+1)(n+2)}\,.$$