

Tutorial 11 Week 12

1. Find a closed form for the generating function of the sequence a_0, a_1, \dots , where $a_0 = 1, a_1 = 1$ and

$$a_n = 5a_{n-1} - 6a_{n-2}.$$

2. Let a_0, a_1, a_2, \dots , be a sequence of integers satisfying the recurrence relation

$$a_n - 2a_{n-1} - a_{n-2} + 2a_{n-3} = 0,$$

for $n \geq 4$, where $a_0 = 0, a_1 = 0, a_2 = 2$ and $a_3 = 5$. Find a closed form for the generating function of this sequence.

3. Solve the following recurrence relations using generating functions:

(i) $x_n = 4x_{n-1} - 3x_{n-2}$, where $x_0 = 1$ and $x_1 = 2$.

(ii) $x_n = 3x_{n-1} - 3x_{n-2} + x_{n-3}$, where $x_0 = 0, x_1 = 1$ and $x_2 = 3$.

4. Consider the recurrence relation

$$a_n = a_{n-1} + 4,$$

for $n > 0$ and where $a_0 = 1$. Find a closed form for its generating function and thereby find a formula for a_n .

Problem Set 11

1. For each of the following recurrence relations, find a closed form for its generating function:

(i) $x_{n+2} - 5x_{n+1} + 6x_n = 0$ for all $n \geq 0$, with $x_0 = 1$ and $x_1 = 3$.

(ii) $a_n - 2a_{n-1} = 4^{n-1}$ for all $n \geq 1$, with $a_0 = 1$.

2. Consider the following recurrence relation

$$x_n = -x_{n-1} + 6x_{n-2},$$

for $n \geq 2$, where $x_0 = 1$ and $x_1 = 1$. Find a closed form for its generating function and then find a formula for x_n .

3. Consider the following recurrence relation

$$a_{n+2} = a_{n+1} + a_n + n$$

for $n \geq 0$, where $a_0 = a_1 = 1$. Write the corresponding generating function in its closed form.

4. (i) Find a closed form for the generating function of the recurrence relation

$$x_n - 5x_{n-1} + 6x_{n-2} = 0, \quad n \geq 2, \quad x_0 = 3, \quad x_1 = 7.$$

- (ii) Find a closed form for the generating function of the recurrence relation

$$x_n = 2x_{n-1} - x_{n-2} + n, \quad n \geq 2, \quad x_0 = 1, \quad x_1 = 2$$

and then find a formula for x_n .