8008A Semester 2 2009

The University of Sydney

Faculties of Arts, Economics, Education and Science

MATH1013

Life Sciences Differential and Difference Equations

November 2009 Lecturers: L Poladian

Time Allowed: One and a Half Hours

Family Name: .....

Other Names: .....

SID: ..... Seat Number: .....

This examination has two sections: Multiple Choice and Extended Answer.

The Multiple Choice Section is worth 50% of the total examination; there are 25 questions; the questions are of equal value; all questions may be attempted.

Answers to the Multiple Choice questions must be entered on the Multiple Choice Answer Sheet.

The Extended Answer Section is worth 50% of the total examination; there are 3 questions; all questions may be attempted; questions are of equal value; all working must be shown.

Calculators will be supplied; no other calculators are permitted.

THE QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM.

Marker's use only	

Notes

Separable Differential Equations:

$$\frac{dx}{dt} = F(t)G(x),$$
$$\int \frac{dx}{G(x)} = \int F(t)\,dt.$$

Linear Difference Equations with constant coefficients:

Consider the second order linear difference equation

$$aX_{n+2} + bX_{n+1} + cX_n = 0.$$

Let r_1 and r_2 be the roots of the auxiliary (or characteristic) equation:

$$ar^2 + br + c = 0.$$

- The general solution has the form

$$X_n = Ar_1^n + Br_2^n,$$

where A and B are arbitrary constants, provided $r_1 \neq r_2$.

Linear Differential Equations with constant coefficients:

Consider the second order linear differential equation

$$a\frac{d^2X}{dt^2} + b\frac{dX}{dt} + cX = 0.$$

Let k_1 and k_2 be the roots of the auxiliary (or characteristic) equation:

$$ak^2 + bk + c = 0.$$

- If the discriminant is positive, $b^2 - 4ac > 0$: the general solution has the form

$$X(t) = Ae^{k_1 t} + Be^{k_2 t}$$

where A and B are arbitrary constants.

- If the discriminant is negative, $b^2 - 4ac < 0$: the general solution has the form

$$X(t) = e^{kt}[A\sin(\omega t) + B\cos(\omega t)]$$

where A and B are arbitrary constants and

$$k = -\frac{b}{2a} \quad \text{and} \quad \omega = \frac{\sqrt{|b^2 - 4ac|}}{2a}.$$

Exponentials and logarithms:

$$\begin{aligned}
e^0 &= 1 \\
e^{a+b} &= e^a e^b \\
e^{\ln(x)} &= x, \qquad x > 0 \\
e^{a\ln(x)} &= x^a, \qquad x > 0 \\
\ln(1) &= 0 \\
\ln(ab) &= \ln(a) + \ln(b), \qquad a > 0, b > 0 \\
\ln(x^a) &= a\ln(x), \qquad x > 0 \\
\ln(e^x) &= x
\end{aligned}$$

Derivatives:

$f(t)$	$f'(t)$
t^n	$n\,t^{n-1}$
e^{kt}	$k\,e^{kt}$
$\ln(kt)$	$\frac{1}{t}$
$\sin(kt)$	$k\,\cos(kt)$
$\cos(kt)$	$-k\,\sin(kt)$
$g(t)h(t)$	$g'(t)h(t) + g(t)h'(t)$
$\frac{g(t)}{h(t)}$	$\frac{g'(t)h(t) - g(t)h'(t)}{h(t)^2}$

Integrals:

$f(t)$	$\int f(t)\,dt$
t^n	$\frac{t^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{at+b}$	$\frac{1}{a}\ln\lvert at+b\rvert + C, \quad a \neq 0$
e^{kt}	$\frac{1}{k}e^{kt} + C, \quad k \neq 0$
$\sin(kt)$	$-\frac{1}{k}\cos(kt) + C, \quad k \neq 0$
$\cos(kt)$	$\frac{1}{k}\sin(kt) + C, \quad k \neq 0$

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