8009A Semester 2 2006

The University of Sydney

Faculties of Arts, Economics, Education,
Engineering and Science

MATH1014

Introduction to Linear Algebra

November 2006

Lecturers: N R O'Brian
L Paunescu

Time Allowed: **One and a half hours**

Name: ....

SID: .... Seat Number: ....

This examination has two sections: Multiple Choice and Extended Answer.

The Multiple Choice Section is worth 25% of the total examination;
there are 15 questions; the questions are of equal value;
all questions may be attempted.

Answers to the Multiple Choice questions must be coded onto
the Multiple Choice Answer Sheet.

The Extended Answer Section is worth 75% of the total examination;
there are 5 questions; each is worth 10 marks;
marks for each part are indicated;
all questions may be attempted;
working must be shown;
Calculators will be supplied.

THE QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM.

Extended Answer Section

Answer these questions in the answer book(s) provided.
Ask for extra books if you need them.

MARKS

1. (a) Find the vector equation of the line in $\mathbb{R}^2$ which passes through the point $P = (2, -1)$ and is perpendicular to the line with general equation $2x - 3y = 1$. **5**

(b) Find the coordinates of the point in $\mathbb{R}^3$ where the line through the points $P = (1, 2, 0)$ and $Q = (0, 1, -1)$ meets the plane $x - y + 2z = 0$. **5**

2. (a) (*i*) Find the general solution of the system of linear equations

$$\begin{array}{cccc} 2x & -y & -4z & = 2 \\ x & -y & -3z & = 1 \end{array}$$

3

(*ii*) Hence find the vector equation of the line of intersection of the two planes described by these equations. **2**

(b) Use the Gauss-Jordan method to find the inverse of $A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 3 & -3 \\ -2 & 3 & 3 \end{bmatrix}$. **5**

3. *Not relevant.*

4. The weather at a particular location is modelled by a Markov chain with two states, wet and dry. The probability that tomorrow will be wet is 0.662 if today is wet and 0.250 if today is dry.

(a) Write down the transition matrix for this Markov chain. **4**

(b) If Monday is dry, what is the probability that the following Wednesday will be wet? **3**

(c) In the long run what will be the proportions of wet and dry days? **3**

5. The females of a particular insect species live a maximum of two years. In her first year each female produces an average of 0.5 female offspring. In the second year each female produces 1 female offspring on average. Half of the females survive their first year to breed in the second year.

(a) Write down the Leslie matrix L for the female insect population. **3**

(b) Calculate the eigenvalues and eigenvectors of L. **3**

(c) Use your answer to the previous part to show the existence of a *steady state* population distribution. **2**

(d) What are the relative proportions of females in their first and second years in this stable population? **2**

End of Extended Answer Section