

Assignment

MATH1014: Introduction to Linear Algebra

Semester 2, 2009

This assignment is due by 4pm on Thursday 22 October.

It should be posted in the glass-fronted collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue. (NOT the glass-fronted collection boxes near the pyramids on Carslaw Level 3, nor the open wooden pigeonholes.) Please do not post your assignment before 22 October, since the boxes are also used for the collection of assignments in other units. Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. See pages 26 and 27 of the Junior Mathematics and Statistics Handbook for further details on assignment submission. A cover sheet must be signed and attached.

1. Let l be the line defined by the parametric equations

$$\begin{aligned}x &= 3 + 2t \\y &= -1 + t \\z &= 5 - 3t.\end{aligned}$$

Let m be the line defined by the vector equation $\mathbf{r} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$,

where $\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Let P be the plane defined by the Cartesian equation $2x - 3y + 5z = 6$.

- Determine whether or not the point $(7, -1, -1)$ lies on l .
- Determine whether or not l and m are parallel.
- Determine whether or not l is perpendicular to the plane P .
- Determine whether or not m lies in P .
- Find the point of intersection of l with P .

2. Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$.

- Find the eigenvalues of A .
- In part (a) you should have found that exactly one of the eigenvalues of A is an integer.
Find the eigenvectors of A corresponding to that eigenvalue.

3. The coat of a certain breed of dog can be either black, white or grey.

When a black-coated male is crossed with a white-coated female, the offspring puppy has a 50% chance of having a black coat, and a 50% chance of having a white coat.

When a white-coated male is crossed with a white-coated female, the offspring puppy has a 20% chance of having a black coat, a 60% chance of having a white coat and a 20% chance of having a grey coat.

When a grey-coated male is crossed with a white-coated female, the offspring puppy has a 10% chance of having a black coat, a 45% chance of having a white coat and a 45% chance of having a grey coat.

- (a) Assume that each successive generation of dogs is produced by crossing all dogs with a white-coated female. Write down the transition matrix for this Markov process.
- (b) If initially there are equal numbers of black-coated and white-coated dogs, and no grey-coated dogs, what percentage of the next generation will be grey-coated?
- (c) In the long run, what percentage of dogs will have black, white and grey coats?

4. The University of Sydney student identification number (SID) is a 9-digit code with check vector $\mathbf{c} = [9, 8, 7, 6, 5, 4, 3, 2, 1]$. A valid SID \mathbf{v} has $\mathbf{c} \cdot \mathbf{v} = 0$ in \mathbb{Z}_{11} .

A common error when entering an SID on machine-readable forms is to enter 0 instead of 9, or $x + 1$ instead of x for $0 \leq x \leq 8$. (So a 0 gets entered as 1, or 1 as 2, for example.)

Prove that a single error of this type will always be detected. (That is, prove that if a single error of this type is made, and the resulting incorrect SID is \mathbf{v} , then $\mathbf{c} \cdot \mathbf{v} \neq 0$ in \mathbb{Z}_{11} .)