THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Tutorial 1 (Week 1)

MATH1014: Introduction to Linear Algebra — Semester 2, 2009

Preparatory questions (attempt before the tutorial)

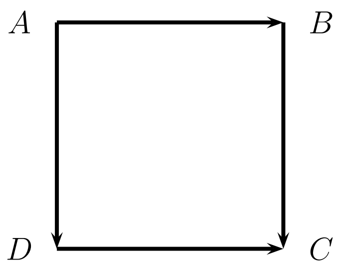
1. If P is the point $(-2, 1)$ in the Cartesian plane, and O is the origin $(0, 0)$, draw the vector $\mathbf{v} = \overrightarrow{OP}$ as a position vector in the plane.

Redraw the vector $\mathbf{v}$ with
(a) its tail at the point $(2, 3)$, and
(b) its head at the point $(-1, -1)$.

2. Suppose $\mathbf{v} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Find each of the following (as a column vector).

(a) $\mathbf{v} + \mathbf{w}$ (b) $3\,\mathbf{v}$ (c) $-2\,\mathbf{w}$ (d) $\mathbf{v} - 5\,\mathbf{w}$

3. The edges of the square $ABCD$ are marked by vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{AD}$, $\overrightarrow{DC}$, as shown.

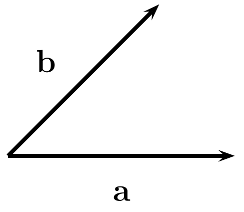


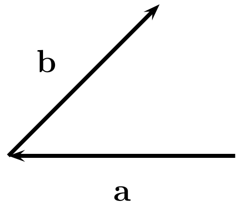
True or false:

(a) $\overrightarrow{AB} = \overrightarrow{BC}$ (b) $\overrightarrow{AB} = \overrightarrow{CD}$ (c) $\overrightarrow{AD} = \overrightarrow{BC}$ (d) $\overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{DC}$

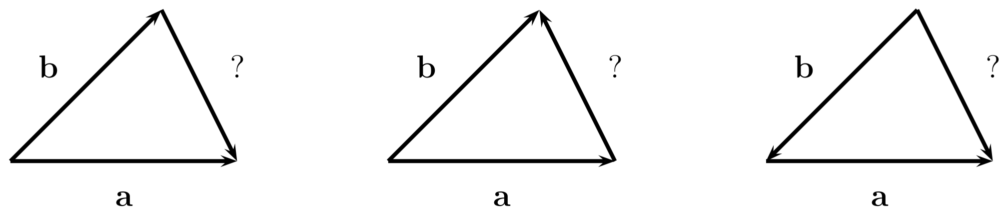
Tutorial exercises

4. Draw the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ on each diagram.





5. In each diagram below, find the unknown vector in terms of **a** and **b**.



6. Let $P = (-2, 1)$, $Q = (2, 2)$, $R = (1, -3)$ and $O = (0, 0)$ be points in the plane.

(a) Draw a sketch showing the vectors $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$, $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{PR}$.

(b) Write each of the vectors in part (a) as a column vector.

(c) Using the column vectors in part (b), verify that $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

7. Simplify the following vector expressions.

(a) $3\mathbf{a} + 2\mathbf{b} - 4(\mathbf{b} + \frac{1}{2}\mathbf{a})$ (b) $-(\mathbf{w} - 6\mathbf{z}) - 2\mathbf{w} + \mathbf{v} - 2\mathbf{z}$

8. Let $ABCDEF$ be a regular hexagon and put

$$\mathbf{a} = \overrightarrow{AB}\,, \qquad \mathbf{b} = \overrightarrow{BC}\,.$$

Find vector expressions in terms of **a** and **b** for the displacements

$$\overrightarrow{CD}, \quad \overrightarrow{DE}, \quad \overrightarrow{EF}, \quad \overrightarrow{FA}.$$

9. Let $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$.

(a) Find $-\mathbf{a}$, $3\,\mathbf{b}$, $\mathbf{a} + 2\,\mathbf{b}$, $3\,\mathbf{a} - 5\,\mathbf{b}$.

(b) If the vector **a** is drawn in $\mathbb{R}^3$ with its head at the point $(0, 0, 0)$, where is its tail?

(c) If the vector **b** is drawn in $\mathbb{R}^3$ with its tail at the point $(-1, 0, 7)$, where is its head?

10. Let $P = (2, -1, 3)$, $Q = (0, 4, 5)$ and $R = (-1, 0, -6)$ be points in $\mathbb{R}^3$.
Let $\mathbf{u} = \overrightarrow{PQ}$, $\mathbf{v} = \overrightarrow{QR}$ and $\mathbf{w} = \overrightarrow{PR}$. Write $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ as column vectors.

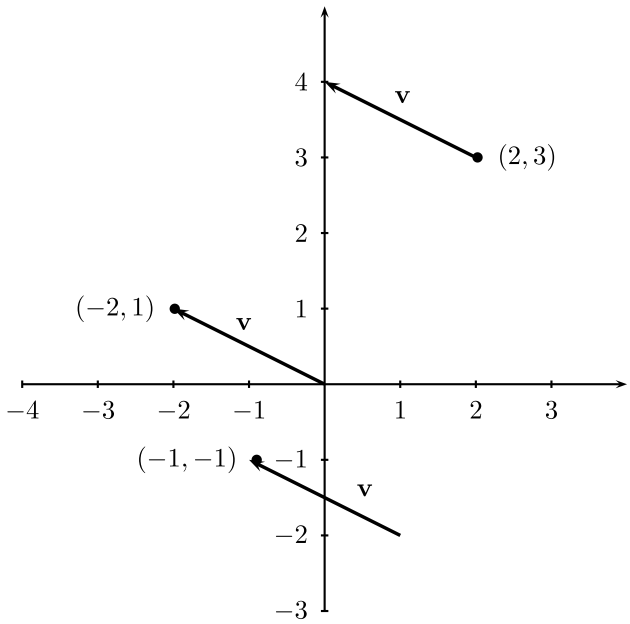
Further exercises

In addition to these exercises, the following exercises from the textbook – *Linear Algebra: A Modern Introduction* by David Poole – should be attempted:

Exercises 1.1:
1, 3, 5, 7, 9, 11, 13, 15, 17.

Solutions

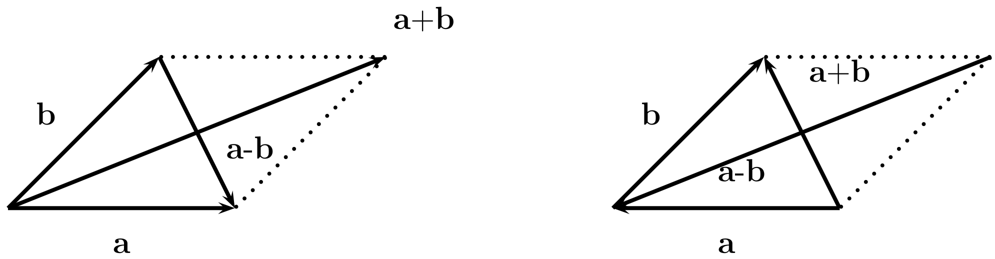
1.



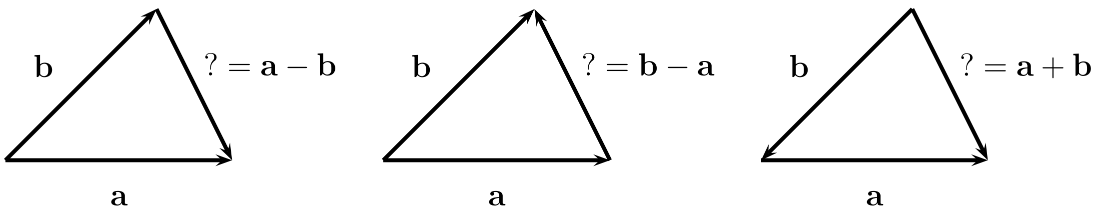
2. (a) $\begin{bmatrix}3\\4\end{bmatrix}$ (b) $\begin{bmatrix}15\\0\end{bmatrix}$ (c) $\begin{bmatrix}4\\-8\end{bmatrix}$ (d) $\begin{bmatrix}15\\-20\end{bmatrix}$

3. (a) false (b) false (c) true (d) true

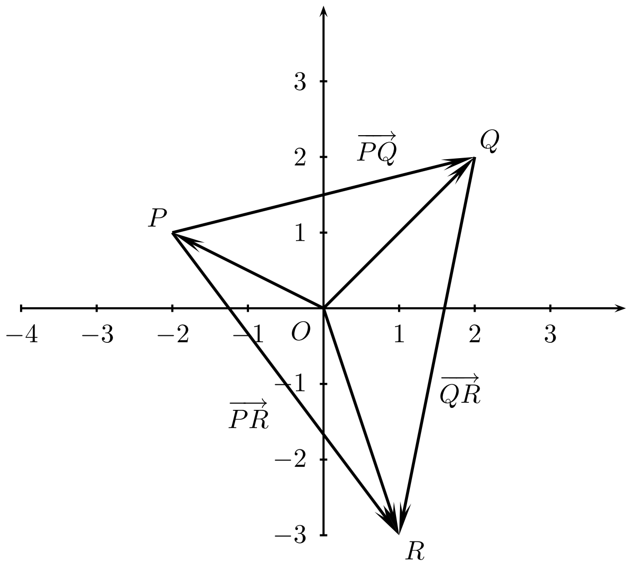
4.



5.



6. (a)



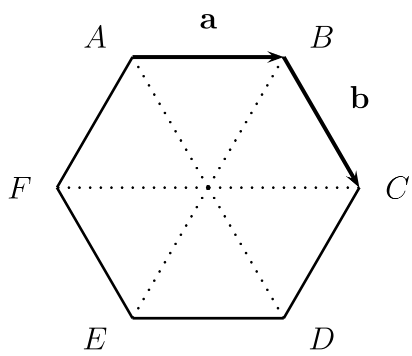
(b) $\overrightarrow{OP} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\overrightarrow{OR} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\overrightarrow{PQ} = \begin{bmatrix} 2-(-2) \\ 2-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$,

$\overrightarrow{QR} = \begin{bmatrix} 1-2 \\ -3-2 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$ and $\overrightarrow{PR} = \begin{bmatrix} 1-(-2) \\ -3-1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$.

(c) $\overrightarrow{PQ} + \overrightarrow{QR} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} 4-1 \\ 1-5 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \overrightarrow{PR}$

7. (a) $\mathbf{a} - 2\mathbf{b}$ (b) $\mathbf{v} - 3\mathbf{w} + 4\mathbf{z}$

8.



$$\overrightarrow{CD} = \mathbf{b} - \mathbf{a}\,, \quad \overrightarrow{DE} = -\mathbf{a}\,, \quad \overrightarrow{EF} = -\mathbf{b}\,, \quad \overrightarrow{FA} = \mathbf{a} - \mathbf{b}\,.$$

9. (a) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 12 \\ -6 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 9 \\ -2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -17 \\ 16 \\ 9 \end{bmatrix}.$

(b) At the point $(-1, -2, -3)$. (c) At the point $(3, -2, 7)$.

10. $\mathbf{u} = \begin{bmatrix} 0-2 \\ 4-(-1) \\ 5-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1-0 \\ 0-4 \\ -6-5 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ -11 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -1-2 \\ 0-(-1) \\ -6-3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -9 \end{bmatrix}$.