The University of Sydney
School of Mathematics and Statistics

Tutorial 3 (Week 3)

MATH1014: Introduction to Linear Algebra — Semester 2, 2009

Preparatory questions (attempt before the tutorial)

1. (a) Find a vector parallel to each of the following straight lines. (Hint: choose any two points on the line and find the vector joining them.)

(i) $y = 2x$ (ii) $x = 2y$ (iii) $2x + y = 0$ (iv) $x + 2y = 0$
(v) $x + y = 2$ (vi) $y = 2x - 3$ (vii) $y = 3 - 2x$ (viii) $2y = x + 5$
(ix) $2y = 5 - x$ (x) $x - 2y = 3$ (xi) $2x + y + 1 = 0$

(b) Which of the lines in part (a) are parallel to the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

Draw a sketch to illustrate your answer.

(c) Which of the lines in part (a) are perpendicular to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

2. (a) Find a vector normal to the plane $x + y + 5z = 6$.

(b) Find a vector normal to the plane $z = 7 - 3x + 8y$.

(c) Find a unit vector normal to the plane $2x + y - z = 6$.

Tutorial exercises

3. Find a vector equation, parametric equations, and a Cartesian equation for the straight line through the point $(0, 0)$ parallel to the vector $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

4. Find a vector equation, parametric equations, and a Cartesian equation for the straight line through the point $(1, 4)$ parallel to the vector $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

5. Find a vector equation, and parametric equations, for the line passing through the point $(1, 0, -1)$ in the direction of the vector $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

6. Find a vector equation, and parametric equations, for the line passing through $P = (-4, 3, 5)$ and $Q = (-2, 4, -1)$.

7. A plane $\mathcal{P}$ contains the point $(2, 3, 5)$ and has normal vector $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$. Find the

equation of $\mathcal{P}$ in normal form, and hence find a Cartesian equation for $\mathcal{P}$.

8. Find the Cartesian equation of the plane containing the points $P = (1, 2, 3)$, $Q = (-1, -2, -3)$ and $R = (4, -4, 4)$.

9. Find parametric equations for the line passing through $(1, 0, -2)$ and perpendicular to the plane $3x - 4y + z = 6$.

10. Show that the line with parametric equations

$$x = 3 + 2t, \quad y = 4 + 3t, \quad z = 5 + 4t \quad (t \in \mathbb{R})$$

is parallel to the plane $4x + 4y - 5z = 14$.

Further exercises

In addition to these exercises, the following exercises from the textbook – *Linear Algebra: A Modern Introduction* by David Poole – are relevant:

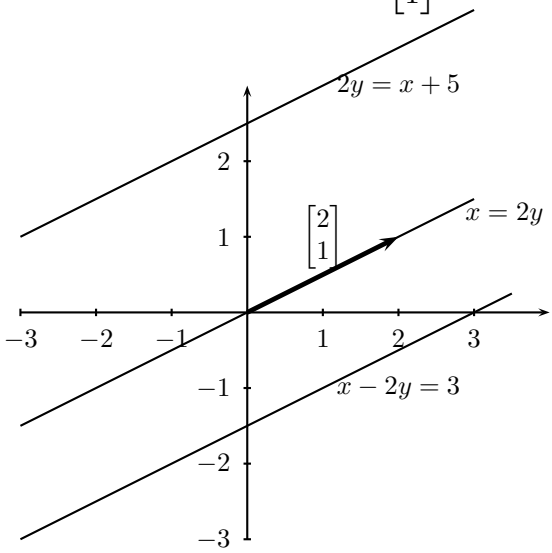
Exercises 1.3: 3, 5, 7, 11, 13, 15, 19, 21, 23.

Answers to selected exercises

1. (a) (i) $\begin{bmatrix}1\\2\end{bmatrix}$ (ii) $\begin{bmatrix}2\\1\end{bmatrix}$ (iii) $\begin{bmatrix}1\\-2\end{bmatrix}$ (iv) $\begin{bmatrix}-2\\1\end{bmatrix}$ (v) $\begin{bmatrix}-2\\2\end{bmatrix}$ (vi) $\begin{bmatrix}2\\4\end{bmatrix}$
(vii) $\begin{bmatrix}1\\-2\end{bmatrix}$ (viii) $\begin{bmatrix}6\\3\end{bmatrix}$ (ix) $\begin{bmatrix}-4\\2\end{bmatrix}$ (x) $\begin{bmatrix}-2\\-1\end{bmatrix}$ (xi) $\begin{bmatrix}1\\-2\end{bmatrix}$

Note: In each case, any multiple of the vector given is also a correct answer.

(b) The lines in (ii), (viii) and (x) are parallel to $\begin{bmatrix}2\\1\end{bmatrix}$.



(c) The lines in (iii), (vii) and (xi) are perpendicular to $\begin{bmatrix}2\\1\end{bmatrix}$.

2. (a) $\begin{bmatrix}1\\1\\5\end{bmatrix}$ (b) $\begin{bmatrix}3\\-8\\1\end{bmatrix}$ (c) $\begin{bmatrix}\frac{2}{\sqrt{6}}\\\frac{1}{\sqrt{6}}\\-\frac{1}{\sqrt{6}}\end{bmatrix}$

3. Vector equation: $\mathbf{x} = \begin{bmatrix}x\\y\end{bmatrix} = t\begin{bmatrix}5\\2\end{bmatrix}, \quad (t \in \mathbb{R}).$

Parametric equations: $x = 5t,\ y = 2t, \quad (t \in \mathbb{R}).$

Cartesian equation: $5y = 2x.$

4. Vector equation: $\mathbf{x} = \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\4\end{bmatrix} + t\begin{bmatrix}5\\2\end{bmatrix}, \quad (t \in \mathbb{R}).$

Parametric equations: $x = 1 + 5t,\ y = 4 + 2t, \quad (t \in \mathbb{R}).$

Cartesian equation: $5y = 2x + 18.$

5. A vector equation: $\mathbf{x} = \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\-1\end{bmatrix} + t\begin{bmatrix}2\\2\\-1\end{bmatrix}, \quad (t \in \mathbb{R}).$

Parametric equations: $x = 1 + 2t, \quad y = 2t, \quad z = -1 - t, \quad (t \in \mathbb{R}).$

6. Vector equation: $\mathbf{x} = \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-4\\3\\5\end{bmatrix} + t\begin{bmatrix}2\\1\\-6\end{bmatrix}, \quad (t \in \mathbb{R}).$

Parametric equations: $x = -4 + 2t, \quad y = 3 + t, \quad z = 5 - 6t, \quad (t \in \mathbb{R}).$

7. Cartesian equation: $x + 3y - z = 6.$

8. $5x + 2y - 3z = 0.$

9. $x = 1 + 3t, \quad y = -4t, \quad z = -2 + t \quad (t \in \mathbb{R}).$