The University of Sydney
School of Mathematics and Statistics

Tutorial 4 (Week 4)

MATH1014: Introduction to Linear Algebra | Semester 2, 2009

Preparatory question (attempt before the tutorial)

1. (a) Write out the addition and multiplication tables for $\mathbb{Z}_5$.

(b) What is 3^{1000} in $\mathbb{Z}_5$?

Tutorial exercises

(*Starred exercises are more difficult. Do not be concerned if you are unable to do them.*)

2. Find $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}\cdot\mathbf{v}$ if $\mathbf{u}$ and $\mathbf{v}$ are the binary vectors $\mathbf{u} = \begin{bmatrix}1\\1\\1\\0\\0\\1\end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix}1\\0\\1\\1\\1\\0\end{bmatrix}$.

3. What is the parity check code vector for each of the following binary vectors?

(a) $[0, 1, 0]$ (b) $[0, 1, 0, 1, 1, 1]$

4. The following parity check code vectors are received. Has there been an error in transmission?

(a) $[1, 0, 0, 1, 1]$ (b) $[1, 0, 0, 0, 0]$ (c) $[1, 0, 1, 0]$

5. In $\mathbb{Z}_3$ calculate

(a) $2 + 2 + 2 + 1$ (c) $2 \times (2 + 2 + 2 + 1)$

(b) $2 + 2 + 2 + 2 + 1$ (d) 2^{10}

6. Perform the following calculations:

(a) $3 \times 4 + 6 \times 5$ in $\mathbb{Z}_7$

(b) $(8 + 7) \times (9 + 4)$ in $\mathbb{Z}_{10}$

(c) $\begin{bmatrix}3\\2\\1\end{bmatrix} \cdot \begin{bmatrix}3\\3\\3\end{bmatrix}$ in $\mathbb{Z}_4^3$ (d) $\begin{bmatrix}3\\4\\1\\2\end{bmatrix} \cdot \begin{bmatrix}3\\3\\4\\4\end{bmatrix}$ in $\mathbb{Z}_5^4$

7. Solve the following equations in $\mathbb{Z}_6$:

(a) $x + 4 = 1$ (b) $2x + 2 = 0$ (c) $x^2 + x = 0$

8. The barcode used on products in Australia is generally a 13-digit Global Trade Item Number, known as a GTIN-13. Each digit is a number between 0 and 9 (that is, an element of $\mathbb{Z}_{10}$). The first 12 digits give information about the product, and the 13th digit is a check digit.
The check vector is $\mathbf{c} = [1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1]$. The check digit is chosen so that the dot product of $\mathbf{c}$ and the barcode vector is 0 in $\mathbb{Z}_{10}$.

(a) Determine whether or not 8 712000 075302 is a valid GTIN-13.

(b) Determine whether or not 40 07817 31056 4 is a valid GTIN-13.

(c) Find the check digit, d, for the GTIN-13: 9 312345 67890 d.

(d) Given that there is an error in the 8th digit (counting from the left) in the following GTIN-13, and that all other digits are correct, determine the correct GTIN-13: 9 310088 801846.

(e) ** Prove that if two adjacent entries in a GTIN-13 differ by 5, and are mistakenly transposed, the error will not be detected.

9. Before 2007, ISBN numbers contained 10 digits, with the tenth digit being a check digit. For 10-digit ISBNs, the check vector is $\mathbf{c} = [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]$, and the check digit is chosen so that the dot product of $\mathbf{c}$ and the ISBN is 0 in $\mathbb{Z}_{11}$.

(a) Determine whether or not the following is a valid ISBN: 0–13–087304–7

(b) Find the check digit, d, in the ISBN: 0–471–16362–d

(c) Find the check digit, d, in the ISBN: 1–86451–323–d

(d) (i) Show that 0–8044–9257–X is not a valid ISBN.

(ii) ** Given that the only error in the number in part (i) is the transposition of an adjacent pair of entries, is it possible to find the correct ISBN?

Further exercises

In addition to these exercises, the following exercises from the textbook – *Linear Algebra: A Modern Introduction* by David Poole – are relevant:

Exercises 1.4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53.

Answers to selected exercises

1. (a)

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(b) We can approach this question by looking for a small power of 3 that is equal to 1 in $\mathbb{Z}_5$. From the multiplication table in part (a), we see that $3^2 = 4$ and $4^2 = 1$, so $3^4 = (3^2)^2 = 4^2 = 1$.
Hence $3^{1000} = (3^4)^{250} = 1$.

2. $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u} \cdot \mathbf{v} = 1 + 1 = 0.$

3. (a) $[0, 1, 0, 1]$ (b) $[0, 1, 0, 1, 1, 1, 0]$

4. (a) and (b) Yes. (c) No.

5. (a) 1; (b) 0; (c) 2; (d) 1.

6. (a) 0 (b) 5 (c) 2 (d) 3

7. (a) $x = 3$ (b) $x = 2,\ 5$ (c) $x = 0,\ 2,\ 3,\ 5$

8. (a) Invalid. (b) Valid. (c) $d = 7$. (d) 9 310088 001846

9. (a) Valid. (b) $d = 7$. (c) $d = 3$.