

Tutorial 6 (Week 6)

MATH1014: Introduction to Linear Algebra

Semester 2, 2009

Preparatory questions (attempt before the tutorial)

1. Which of the following matrices are in row echelon form?

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(i) $\begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(j) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(k) $\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(h) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(l) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

2. Reduce the following matrices to row echelon form.

(a) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -4 & -2 & 6 \\ 3 & 1 & 6 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$

Tutorial exercises

3. Solve the following systems of equations by writing down the associated augmented matrix and row reducing.

(a)
$$\begin{aligned} x + y - z &= 0 \\ 2x - y + z &= 9 \\ x + z &= 10 \end{aligned}$$

(d)
$$\begin{aligned} x - y - z &= 0 \\ x - 2y + z &= 0 \\ 2x + z &= 0 \end{aligned}$$

(b)
$$\begin{aligned} -3x + 2y + z &= 4 \\ 4x + y + 3z &= 9 \\ x - y - z &= -4 \end{aligned}$$

(e)
$$\begin{aligned} x + 2y + 7z &= 5 \\ x + y + 4z &= 3 \\ 2x + 3y + 11z &= 7 \end{aligned}$$

(c)
$$\begin{aligned} x + 2y + 3z &= 0 \\ 3x + 2y + z &= 0 \end{aligned}$$

(f)
$$\begin{aligned} x + 2y + z - w &= 4 \\ 2x + 4y - z + 4w &= -1 \\ -x - 2y + 2z - 5w &= 5 \end{aligned}$$

4. Interpret the solutions to Question 3 (a), (c), (d) and (e) geometrically.

5. Solve the following systems of equations.

$$\begin{aligned} \text{(a)} \quad & x - y + z = 0 \\ & -x + 3y + z = 5 \\ & 3x + y + 7z = 2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & -x_1 + 3x_2 - 2x_3 + 4x_4 = 0 \\ & 2x_1 - 6x_2 + x_3 - 2x_4 = -3 \\ & x_1 - 3x_2 + 4x_3 - 8x_4 = 2 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & a + b + c + d = 4 \\ & a + 2b + 3c + 4d = 10 \\ & a + 3b + 6c + 10d = 20 \\ & a + 4b + 10c + 20d = 35 \end{aligned}$$

6. Find the values of k (if any) such that the following system has (i) no solution; (ii) a unique solution; (iii) infinitely many solutions.

$$\begin{aligned} x - 2y + 3z &= 2 \\ x + y + z &= k \\ 2x - y + 4z &= k^2 \end{aligned}$$

7. Find the values of k (if any) such that the following system (i) is inconsistent; (ii) has infinitely many solutions; (iii) has a unique solution:

$$\begin{aligned} x & - 3z = -3 \\ -2x - ky + z &= 2 \\ x + 2y + kz &= 1 \end{aligned}$$

Further exercises

In addition to these exercises, the following exercises from the textbook – *Linear Algebra: A Modern Introduction* by David Poole – are relevant:

Exercises 2.2: 1, 3, 5, 7, 9, 11, 13, 25, 27, 29, 31, 33, 35, 37, 41, 43, 45.

Answers to selected exercises

1. The matrices in (a), (c), (e), (f), (h), (i) and (j) are in row echelon form. The others are not.

2. (a) Perform the following elementary row operations, in order:

$$R1 \rightarrow R1 \div 4, \quad R2 \rightarrow R2 - 2R1, \quad R2 \rightarrow R2 \times -2$$

The resulting row echelon form is $\begin{bmatrix} 1 & 3/4 \\ 0 & 1 \end{bmatrix}$.

- (b) Perform the following elementary row operations, in order:

$$R1 \rightarrow R1 \div 2, \quad R2 \rightarrow R2 - 3R1, \quad R2 \rightarrow R2 \div 7$$

The resulting row echelon form is $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 9/7 & -3/7 \end{bmatrix}$.

- (c) Perform the following elementary row operations, in order:

$$R1 \leftrightarrow R3, \quad R2 \rightarrow R2 + 3R1, \quad R3 \rightarrow R3 + 2R1, \quad R3 \rightarrow R3 - R2$$

The resulting row echelon form is $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

3. (a) $x = 3, y = 4, z = 7$.
(b) $x = -1, y = -2, z = 5$.
(c) $x = t, y = -2t, z = t$.
(d) $x = y = z = 0$.
(e) No solutions.
(f) $x = 1 - 2s - t, y = s, z = 3 + 2t, w = t$.
5. (a) No solutions.
(b) $x_1 = -2 + 3s, x_2 = s, x_3 = 1 + 2t, x_4 = t$.
(c) $a = b = c = d = 1$.
6. (i) $k \neq 2, k \neq -1$.
(ii) No values of k .
(iii) $k = 2$ or -1 .
7. (i) $k = -5$.
(ii) $k = 2$.
(iii) $k \neq 2, k \neq -5$.