The University of Sydney
School of Mathematics and Statistics

Tutorial 8 (Week 8)

MATH1014: Introduction to Linear Algebra — Semester 2, 2009

Preparatory questions (attempt before the tutorial)

1. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$, then $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

(a) Use this formula to find the inverse of each of the following matrices, if it exists:

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix},\quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\quad C = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix},\quad D = \begin{bmatrix} 5 & 4 \\ 3 & 3 \end{bmatrix},\quad E = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}.$$

(b) Verify that: $AA^{-1} = I_2$, $B^{-1}B = I_2$, $DD^{-1} = I_2$, $E^{-1}E = I_2$.

(I_2 is the 2×2 identity matrix, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.)

Tutorial exercises

2. Find the inverse of $\begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$.

3. Find the inverse of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.

(a) Find AB, A^{-1}, and B^{-1}.

(b) Find $B^{-1}A^{-1}$. Calculate the products $(AB)(B^{-1}A^{-1})$ and $(B^{-1}A^{-1})(AB)$. What do these products tell you about $(AB)^{-1}$?

5. Use B^{-1} from Question **4** to solve

(a)
$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x + 3y + z &= 0 \\ 3x + y + 2z &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + 3z &= 2 \\ 2x + 3y + z &= 2 \\ 3x + y + 2z &= 4 \end{aligned}$$

6. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & -1 \\ -3 & 5 & 3 \end{bmatrix}$.

(a) Find AB and BA.

(b) Does A have an inverse? Does B have an inverse?

7. A square matrix in which all the entries off the main diagonal are zero is called a *diagonal matrix*.

(a) Let A be the diagonal matrix $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Find A^{-1}.

(b) The general $n \times n$ diagonal matrix is:

$$D = \begin{bmatrix} d_1 & 0 & \dots & 0 \\ 0 & d_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_n \end{bmatrix}$$

Explain why D is not invertible if $d_i = 0$ for some $i = 1, 2, \dots, n$.
When all the diagonal entries are non-zero, what is D^{-1}?

Further exercises

In addition to these exercises, the following exercises from the textbook – *Linear Algebra: A Modern Introduction* by David Poole – should be attempted:

Exercises 3.3: 1, 3, 5, 7, 9, 11, 49, 51, 55.

Answers to selected exercises

1. (a) $A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, $B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, C^{-1} does not exist, $D^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -4 \\ -3 & 5 \end{bmatrix}$,

$E^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$.

2. $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{bmatrix}$.

3. There is no inverse.

4. (a) $AB = \begin{bmatrix} -2 & -3 & -1 \\ 7 & 7 & 4 \\ 1 & 2 & 3 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$, $B^{-1} = \frac{1}{18}\begin{bmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{bmatrix}$.

(b) $B^{-1}A^{-1} = \frac{1}{18}\begin{bmatrix} 13 & 7 & -5 \\ -17 & -5 & 1 \\ 7 & 1 & 7 \end{bmatrix}$, $(AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})(AB) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

5. (a) $x = y = z = 0$ is the only solution.

(b) $x = \frac{10}{9}$, $y = -\frac{2}{9}$, $z = \frac{4}{9}$.

6. (a) $AB = \begin{bmatrix} -4 & 10 & 5 \\ 3 & -5 & -3 \\ -10 & 20 & 11 \end{bmatrix}$, $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.