1. Below is a sketch of the curve \( y = e^x \) together with the tangent line to the \( y \)-intercept: \( y = e^x \) with \( y = x \).

Add to the above diagram the reflection of this curve and the tangent in the line \( y = x \).

What is the new curve? \( y = \) __________

What is the slope of the reflected tangent line? Slope = __________
2.

\[ y = \ln x \]

In the above diagram, what does the ratio \( \frac{\ln (1 + \text{th})}{h} \) represent?

Hence evaluate \( \lim_{{h \to 0}} \frac{\ln (1 + \text{th})}{h} = \)

Complete the following:

\[ \frac{d}{dx} (\ln x) = \lim_{{h \to 0}} \frac{\ln (x + h) - \ln x}{h} \]

\[ = \lim_{{h \to 0}} \frac{\ln \left( \frac{x+h}{x} \right)}{h} = \lim_{{h \to 0}} \frac{\ln (1 + \frac{h}{x})}{x \frac{h}{x}} \]

\[ = \frac{1}{x} \left( \lim_{{h \to 0}} \frac{\ln (1 + \frac{h}{x})}{\frac{h}{x}} \right) = \]
3. **Chain Rule**: \[ \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \]

(i) let \( y = e^{-x} \) and put \( u = -x \).

Then \[ \frac{dy}{du} = \frac{d}{du} (e^u) = \]

and \[ \frac{du}{dx} = \]

so \[ \frac{dy}{dx} = \]

(ii) let \( y = e^{kx} \) where \( k \) is a constant.

Put \( u = kx \) so \[ \frac{du}{dx} = \]

Hence \[ \frac{dy}{dx} = \]
4. Let \( y = \ln (2x) \).

Complete the following:

\[
\ln (2x) = \ln 2 + \square
\]

so \( \frac{d}{dx} (\ln (2x)) = \frac{d}{dx} (\ln 2) + \frac{d}{dx} \square \)

= \square

Alternatively, let's use the chain rule.

Put \( u = 2x \) so \( \frac{du}{dx} = \square \)

and

\[
\frac{d}{dx} (\ln (2x)) = \frac{d}{du} (\ln (u)) \frac{du}{dx}
\]

= \square

If \( k \) is a positive constant then

\[
\frac{d}{dx} (\ln (kx)) = \square
\]
5. **Product Rule**: if \( f(x) = u \cdot v \)

then \[ f'(x) = \frac{d}{dx}(u \cdot v) = u \frac{dv}{dx} + v \frac{du}{dx} \]

Let \( f(x) = (x+1)(2x-3) \)

Choose \( u = \) \[ \] \, \( v = \) \[ \]

so \( \frac{du}{dx} = \) \[ \] \, \( \frac{dv}{dx} = \) \[ \]

By the Product Rule

\[ f'(x) = \]

\((*)\)

Alternatively, expand the brackets:

\( f(x) = (x+1)(2x-3) = \)

and differentiate the polynomial directly:

\( f'(x) = \)

\((***)\)

Check that \((*)\) and \((***)\) agree.
6. Let \( g(x) = (x^2 + x + 4)(x^3 - 2) \)

Choose \( u, v \) so that \( g(x) = uv \):

\[
  u = \quad , \quad v = \quad
\]

So \( \frac{du}{dx} = \quad , \quad \frac{dv}{dx} = \quad \).

By the Product Rule

\[
g'(x) = \quad
\]

Alternatively, expand the brackets:

\[
g(x) = (x^2 + x + 4)(x^3 - 2)
\]

\[
= \quad
\]

and differentiate the polynomial:

\[
g'(x) = \quad
\]
7. Add the curve \( y = \frac{1}{x} = x^{-1} \) to the following diagram:

\[ \begin{array}{c}
| y \\
\downarrow \\
-1 \\
\uparrow \\
1 \\
\downarrow \\
-1 \\
\uparrow \\
0 \\
\uparrow \\
\rightarrow x \\
\end{array} \]

Complete the following:

\[
\begin{align*}
\lim_{x \to \infty} \frac{1}{x} &= \square, & \lim_{x \to -\infty} \frac{1}{x} &= \square \\
\lim_{x \to 0^+} \frac{1}{x} &= \square, & \lim_{x \to 0^-} \frac{1}{x} &= \square
\end{align*}
\]

\[
\frac{d}{dx} (x^{-1}) = \frac{d}{dx} \left( \frac{1}{x} \right) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}
\]

= \square
8. Let's use the Product Rule to find 
\[
\frac{d}{dx} (x^{-2}) \quad \text{and} \quad \frac{d}{dx} (x^{-3})
\]

Observe \( x^{-2} = \frac{1}{x^2} = \left( \frac{1}{x} \right) \left( \frac{1}{x} \right) \)

so put \( u = \sigma = \frac{1}{x} = x^{-1} \).

Complete the following:

\[
\frac{du}{dx} = \frac{d\sigma}{dx} =
\]

so

\[
\frac{d}{dx} (x^{-2}) = u \frac{d\sigma}{dx} + \sigma \frac{du}{dx}
\]

Observe \( x^{-3} = x^{-2} x^{-1} \)

so now put \( u = x^{-2} \), \( \sigma = x^{-1} \).

Then

\[
\frac{d}{dx} (x^{-3}) = u \frac{d\sigma}{dx} + \sigma \frac{du}{dx}
\]