1. Let \( y = 4x^2 - 4x + 3 \).

Find \( y' = \) \[ \Box \] .

For which \( x \) is \( y' = 0 \) ?

Answer: \( x = \) \[ \Box \]

Draw a sign diagram for \( y' \):

\[
\begin{array}{c|c}
 x & y' \\
 \hline
 & 1 \\
\end{array}
\]

The minimum value for \( y \) is

Sketch the parabola \( y = 4x^2 - 4x + 3 \):

\[
\begin{array}{c}
| y \\
2 \\
4 \\
6 \\
\hline
| x \\
1 \\
2 \\
\end{array}
\]
2. Approach the quadratic from the previous exercise by completing the square:

\[ y = 4x^2 - 4x + 3 \]
\[ = (4x^2 - 4x + 1) + 2 \]
\[ = (2x-1)^2 + 2 \]  \((*)\)

Put \( u = 2x - 1 \), so \( y = u^2 + 2 \).

Then \( \frac{du}{dx} \) and \( \frac{dy}{du} \).

By the Chain Rule,

\[ \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \]

as before.

Use (*) to explain why the minimum value of \( y \) must be 2 when \( x = \frac{1}{2} \), without using the derivative:
3. Let \[ y = 2x^3 - 9x^2 + 12x - 3 \].

Find \[ y' = \]

and \[ y'' = \]

Then \[ y' = 0 \] when \[ x = \square \] and \[ x = \square \]

and \[ y'' = 0 \] when \[ x = \square \].

Draw sign diagrams for \[ y' \] and \[ y'' \]:

\[ x \]
\[ \downarrow \]
\[ y' \]
\[ - \]
\[ + \]
\[ + \]

\[ x \]
\[ \downarrow \]
\[ y'' \]
\[ - \]
3. (continued)
The local maximum occurs when \( x = \) \[ \square \]
and \( y = \) \[ \square \]

The local minimum occurs when \( x = \) \[ \square \]
and \( y = \) \[ \square \]

The inflection occurs when \( x = \) \[ \square \]
and \( y = \) \[ \square \]

Add the curve \( y = 2x^3 - 9x^2 + 12x - 3 \) to the diagram:
4. Consider the curve

\[ y = \tan x = \frac{\sin x}{\cos x} \quad \text{for} \quad -\frac{\pi}{2} < x < \frac{\pi}{2} : \]

(i) Is the function increasing or decreasing?

Answer: 

(ii) Where is the curve concave up?

Answer: 

(iii) Where is the curve concave down?

Answer: 

(iv) The inflection point is \((x, y) = \)

(v) Complete the following:

\[ \lim_{x \to -\frac{\pi}{2}^{-}} \tan x = \quad \lim_{x \to -\frac{\pi}{2}^{+}} \tan x = \]
5. Let \( y = \tan x = \frac{u}{v} \)

where \( u = \sin x \) and \( v = \cos x \).

Then \( \frac{du}{dx} = \), \( \frac{dv}{dx} = \)

so \( y' = \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \)

Use the Chain Rule to find

\( y'' = \)

Complete the sign diagram for \( y'' \) near \( x = 0 \)

\( \begin{array}{c|c|c} x & 0 & 1 \\ \hline y'' & & \\ \end{array} \)

(Confirming that \((0,0)\) is the inflection point of the curve \( y = \tan x \)).
6. Add the curve \( y = \tan^{-1} x \) below:

![Graph of \( y = \tan^{-1} x \)]

(i) Is the function increasing or decreasing?
Answer: 

(ii) Where is the curve concave up?
Answer: 

(iii) Where is the curve concave down?
Answer: 

(iv) The inflection point is \((x, y) = \) 

(v) Complete the following:
\[ \lim_{x \to \infty} \tan^{-1} x = \phantom{\text{□}} \], \[ \lim_{x \to -\infty} \tan^{-1} x = \phantom{\text{□}} \]
7. Let \( y = \tan^{-1} x \), so \( x = \tan y \).

Then \( \frac{dx}{dy} = \) \[\Box\] (in terms of \( y \)).

Hence \( \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \) \[\Box\] (still in terms of \( y \)).

Consider the following right-angled triangle:

\[\begin{array}{c}
\text{h} \\
\text{y} \\
\text{x}
\end{array}\]

Express the following in terms of \( x \):

\( \tan y = \) \[\Box\], \( h = \) \[\Box\]

\( \sin y = \) \[\Box\], \( \cos y = \) \[\Box\]

Now find \( \frac{dy}{dx} \) in terms of \( x \):

\( \frac{dy}{dx} = \) \[\Box\]

(and you obtain the witch of Maria Agnesi!!)