

Solutions to Assignment 1. Due date: Thursday 2 April 2009.

MATH1111: Introduction to Calculus

Semester 1, 2009

This assignment is due by 4pm on Thursday 2 April 2009. It should be posted in the glass-fronted collection boxes on the verandah of Carlsaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue. (NOT the glass-fronted collection boxes near the pyramids on Carlsaw Level 3, nor the open wooden pigeonholes.) Please do not post your assignment before Thursday 2 April 2009, since the boxes are also used for the collection of assignments in other units. Your assignment must be stapled inside a manilla folder, a cover sheet must be signed and attached, and you must write the initial of your family name as a large letter on the front of the folder. See page 26 of the Junior Mathematics and Statistics Handbook for further guidelines re submission of assignments.

1. Consider the function $f(x) = x^2 - 2x - 15$.

(a) Factorise $f(x)$.

Solution: $f(x) = (x - 5)(x + 3)$.

1 Mark

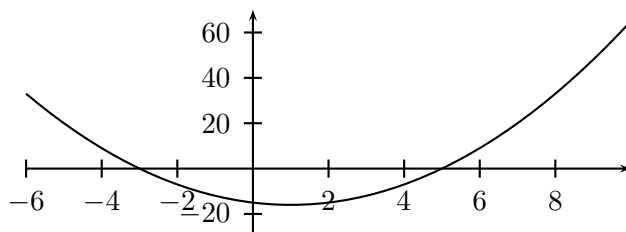
(b) Find the points where the graph of $y = f(x)$ cuts the x -axis.

Solution: When the graph cuts the x -axis, $x^2 - 2x - 15 = 0$. i.e. $(x - 5)(x + 3) = 0$ and $x = 5$ or $x = -3$. So the points are $(5, 0)$ and $(-3, 0)$.

1 Mark

(c) Graph $y = f(x)$, marking on your diagram any points where the curve cuts the x and y axes.

Solution:



2 Marks: 1 for parabola, 1 for correctly marking the x intercepts $(5, 0), (-3, 0)$, and y intercept $(0, -15)$.

2. Find the amplitude and period of $f(t) = -2 \cos(\pi t - 5)$.

Solution: The amplitude is 2. The period is $\frac{2\pi}{\pi} = 2$.

2 Marks: 1 for the period and 1 for the amplitude.

3. Solve $\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ for $0 \leq x \leq \pi$.

Solution: $\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ implies $x + \frac{\pi}{4} = \dots, \frac{-5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \dots$ so $x = \dots, \frac{-3\pi}{2}, 0, \frac{\pi}{2}, \dots$ or in the prescribed domain $x = 0, \frac{\pi}{2}$.

2 Marks: 1 for finding the values of $x + \frac{\pi}{4}$ and 1 for the final solution.

4. Consider the function $f(x) = \frac{x + 1}{x^2 + 2x - 3}$.

- (a) Find the domain of $f(x)$.

Solution: Look for when $x^2 + 2x - 3 = 0$ i.e. when $(x - 1)(x + 3) = 0$. And so the domain is: All x except $x = 1$ and $x = -3$ or $x \in (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.

2 Marks: 2 marks for the correct domain, 1 mark for knowing the denominator can't be zero.

- (b) Does the range include 1? Show your working.

Solution: Consider $f(x) = 1$. This implies $x + 1 = x^2 + 2x - 3 \Rightarrow x^2 + x - 4 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{17}}{2}$. So yes the range includes 1.

2 Marks: 2 marks, 1 mark for knowing this questions is asking if $f(x) = 1$ is possible.

- (c) Find $f(2)$.

Solution: $f(2) = \frac{3}{5}$.

1 Mark

- (d) Find and simplify $f(x - 1)$.

Solution: $f(x - 1) = \frac{x}{(x - 2)(x + 2)} = \frac{x}{x^2 - 4}$.

1 Mark

- (e) Does the graph of $y = f(x)$ cut the x -axis? If so, where?

Solution: $f(x) = 0 \Rightarrow \frac{x + 1}{x^2 + 2x - 3} = 0$. i.e. $x + 1 = 0$ or $x = -1$. So $f(x)$ cuts the x -axis at $(-1, 0)$.

1 Mark

5. Four hours ago a biologist observed the size of a bacteria population grow from a size of 1000 to a size of 1450 in an hour.

- (a) Find an exponential function of the form $P(x) = P_0 e^{kx}$ that models this situation, where $P(x)$ is the size of the bacteria population and x is the number of hours (from when the population was 1000).

Solution: At $x = 0$, $P(0) = P_0 = 1000$. An hour later, $x = 1$ and $P(1) = 1000e^k = 1450$, i.e. $e^k = 1.45$ or $k = \ln(1.45) \approx 0.37$.

2 Marks: 1 for P_0 and 1 for k .

- (b) According to these calculations, what is the current size of the bacteria population?

Solution: Currently $x = 4$ and $P(4) = 1000e^{0.37 \times 4} \approx 4393$.

1 Mark

- (c) When will the original size of the population have grown by a factor of ten?

Solution: When is $C(x) = 10000$? $1000e^{0.37x} = 10000 \Rightarrow e^{0.37x} = 10 \Rightarrow x = \frac{\ln 10}{0.37} \approx 6$. So, it takes 6 hours for the population size to have grown by a factor of ten.

2 Marks: 1 for knowing the equation $C(x) = 10000$ and 1 for the solution.