The University of Sydney
School of Mathematics and Statistics

Solutions to Assignment 2

MATH1111: Introduction to Calculus — Semester 1, 2009

1. Consider $f(x) = -4x^2 + x - 1$.

(a) Find $f'(x)$.

Solution: $f'(x) = -8x + 1$.
1 Mark

(b) Find the point on the graph $y = f(x)$ where $f'(x) = 0$.

Solution: $f'(x) = -8x + 1 = 0 \Rightarrow x = \frac{1}{8}$. And $f(\frac{1}{8}) = -\frac{15}{16}$. So the point for which $f'(x) = 0$ is $(\frac{1}{8}, -\frac{15}{16})$.
2 Marks: 1 for $x = 1/8$ and 1 for the point $(\frac{1}{8}, -\frac{15}{16})$.

(c) Find $f''(x)$.

Solution: $f''(x) = -8$.
1 Mark

(d) Using your result from part c) explain why the graph of $y = f(x)$ is concave down.

Solution: $f(x)$ is concave down as $f''(x) < 0$ for all x.
1 Mark

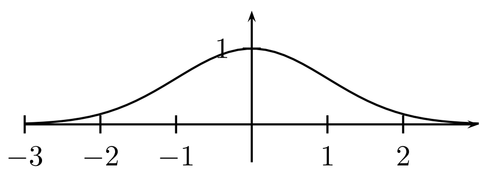
2. Find the equation of the line tangent to the graph of f at $(\pi, 0)$, where f is given by

$$f(x) = x^3 \sin(2x).$$

Solution: $f'(x) = 3x^2 \sin(2x) + 2x^3 \cos(2x)$. So $f'(\pi) = 2\pi^3$ and the equation of the line tangent at $(\pi, 0)$ is $y = 2\pi^3(x - \pi)$.
3 Marks: 1 for $f'(x)$, 1 for $f'(\pi)$ and 1 for the equation of the tangent line.

3. Find all the local maximum, local minimum and inflection points of $f(x) = e^{-x^2/2}$ and hence sketch $y = f(x)$ showing all the points you've found.

Solution: $f'(x) = -xe^{-x^2/2}$ and $f''(x) = -e^{-x^2/2} + (-x)(-x)e^{-x^2/2} = e^{-x^2/2}(x^2 - 1)$.
$f'(x) = 0 \Rightarrow x = 0$. $f''(0) = -1 < 0$ so $(0, 1)$ is a local maximum.
$f''(x) = 0 \Rightarrow x^2 = 1$ or $x = \pm 1$. $f(1) = e^{-1/2}$ and $f(-1) = e^{-1/2}$, so the points of inflection are $(-1, e^{-1/2})$ and $(1, e^{-1/2})$ or approximately $(-1, 0.6)$ and $(1, 0.6)$.
Also as $x \to \infty$, $y \to 0$. And as $x \to -\infty$, $y \to 0$.



8 Marks: 1 for $f'(x)$, 1 for $f'(x) = 0 \Rightarrow x = 0$, 1 for classifying $x = 0$ as yielding a local maximum, 1 for $f''(x)$, 1 for $f''(x) = 0 \Rightarrow x = \pm 1$, 2 for the coordinates of the two points of inflection and 1 for the graph.

4. The number, N, of people who have heard a rumor spread by mass media at time, t, in days, is modelled by

$$N(t) = \frac{a}{1 + be^{-kt}}.$$

(a) If 50 people have heard the rumour initially and 300,000 people hear the rumour eventually, find a and b.

Solution: $N(0) = \frac{a}{1+b} = 50$ and as $t \to \infty$, $N \to 300000$, so $a = 300000$. And so $\frac{300000}{1+b} = 50$ or $b = 5999$.
4 Marks: 1 for $\frac{a}{1+b} = 50$, 1 for $t \to \infty$ implies $N \to a$, 1 for a and 1 for b.

(b) If the rumour is initially spreading at the rate of 500 people per day, find k.

Solution: $N(t) = \dfrac{300000}{1 + 5999e^{-kt}}$, so $N'(t) = \dfrac{300000 \times 5999 \times ke^{-kt}}{(1 + 5999e^{-kt})^2}$.
$N'(0) = \dfrac{300000 \times 5999 \times k}{(6000)^2} = 500$, so $k = \dfrac{60000}{5999} \approx 10$.
3 Marks: 1 for $N'(t)$, 1 for $N'(0)$ and 1 for $k \approx 10$.